Tyre dynamics, tyre as a vehicle component
Part 1.: Tyre handling performance

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1. General

The vehicle handling performance is directly related to the tyre-road contact. The tyres transfer the horizontal and vertical forces acting on the vehicle as a result of steering, braking and driving in combination with possible road disturbances or external disturbances like aerodynamic forces due to for example cross-wind.

The relationship between vehicle behaviour and these tyre-road contact forces depends on the specific tyre design, tyre condition variables like slip and tyre load, the road surface design, and the actual road surface and weather conditions, see figure 1. Tyre design parameters are related to the tyre geometry (width, sidewall height,...), the specific tyre brand (manufacturer), the tread pattern design, the structure of the tyre (a tyre is built up from different rubber compounds and rubberized fabric or cord acting as reinforcement elements, referred to as plies), the amount of wear, etc. More wear may result in a higher stiffness and therefore higher contact forces for the same slip, with slip referring to the deviation of the tyre condition from free rolling conditions (explained more rigorously in the next sections).

![Figure 1.: The tyre-road interface](image-url)
1.1. Effect of tyre ply-design
One may be familiar with the different performance of bias-ply tyres and radial-ply tyres, being a direct result of the different ply-designs of the two tyre types, see figure 2.
For a bias-ply tyre, the belt plies cross over at each other at a large angle (in the order of 40° with respect to the circumferential direction), and are extended over the sidewalls, in contrast to the radial-ply tires with distinction between belt plies (with orientation being close to circumferential) and radial casing plies. Due to these structural differences, the tread motion is reduced for the radial-ply tyre, and the cornering stiffness ('stiffness against cornering', treated in detail later in this chapter) is usually exceeding that for the bias-ply tyre. In figure 3, shear stresses in the contact area along the tyre width are shown, indicating stress concentrations at the tyre shoulders for bias-ply tyres, due to interaction of tread motion and side wall deformation. This contributes to more wear. In addition, bias-ply tyres experience more dissipation, having a positive effect on ride.
1.2. Tyre variables and tyre performance
Tyre load is related to vehicle mass and axle load distribution, and therefore also to loading conditions. Under specific braking, driving or cornering conditions, roll and pitch will occur and tyre loads will change, leading to different response of the tyre-road contact in terms of tyre forces. There is a dependency of tyre-road contact performance on vehicle and tyre forward velocity. Changing the internal tyre pressures will result in a modified contact area and consequently modified local normal pressures in the contact area. This will affect the local shear stress behaviour, building up the horizontal contact forces.

1.3. Road surface parameters
One may distinguish different road surface design in terms of micro- and macrotexture describing the local roughness and adhesion potential, the used materials (asphalt, concrete,..), and the composition of the materials (dense asphalt, drain asphalt,..).
In figure 4, we show some of the effects of the road surface texture on the vehicle and tyre performance (from [33]).

The handling characteristics, i.e. the topic of this chapter, is affected by road texture for wavelengths between values far less then 1 mm, and in the order of 0.5 until 1 meter. Tyre wear is typically a phenomenon related to wavelengths less then 1 cm. Internal vehicle noise is arising from large wavelengths (of course also depending on vehicle speed). Rolling resistance, discomfort and vehicle wear are most affected by the more coarse parts of the road texture as well, and even (discomfort, vehicle wear) by the more global vehicle unevenesses.

![Figure 2.: Functional properties road surface vs. texture wavelength](image-url)
Road conditions vary in time due to ageing. Due to road use, there will be road surface wear and polishing effects, leading to different friction under similar weather conditions. Finally, the weather conditions itself obviously have a strong effect on the tyre-road conditions, where one may think of rain, snow, variation of temperature, and the impact of rain mixing up with dirt after a long dry period leading to significant reduction in road friction.

1.4. Tyre input and output quantities.
A tyre is schematically shown in figure 5, with indication of all the input and output quantities, see also [18]. There are three forces and three moments acting on the tyre:

- \( F_x \): braking, driving force
- \( F_y \): lateral (cornering) force
- \( F_z \): tyre load, to carry the vehicle weight
- \( M_x \): Overturning moment
- \( M_y \): Moment about the wheel axis (driving, braking torque)
- \( M_z \): Self-aligning moment

Most of these forces and moment will be explained later in more detail.

A tyre travels with a horizontal velocity \( V \), with components \( V_x \) and \( V_y \) in longitudinal and lateral direction. Due to brake or drive torque and cornering forces, slip will occur which means that the tyre slides with nonzero speed over the surface. The corresponding slip speeds \( V_{sx} \) and \( V_{sy} \) are shown in figure 5 as well. The tyre rolls over the surface with an angular speed \( \Omega \), leading to the so-called rolling speed:

\[
V_r = \Omega R_e
\]

with \( R_e \) being the effective rolling radius of the tyre under free rolling. For a free rolling wheel, the rolling speed coincides with \( V_x \), defining the effective rolling radius as the ratio between \( V_x \) and \( \Omega \).

1.4.1. The effective rolling radius
The effective rolling radius is not the same as the loaded tyre radius \( R_l \), with the latter being defined as the vertical distance between the wheel centre and the horizontal surface. A free rolling tyre rotates around a point near the contact patch. For a rigid wheel on a flat horizontal surface, this point coincides with the single contact point between tyre and road, and the forward speed \( V_x \) equals angular speed time (loaded = unloaded) radius.
For a pneumatic tyre, the distance between points at the circumference of the tyre and the wheel centre varies from a value close to the unloaded radius just before entering the contact area to the same value as the loaded radius just at the projection point of the wheel centre on the contact area. At that point, the peripheral velocity of the tread (relative to the wheel centre) coincides with the horizontal velocity \( V \) of the wheel centre. Moving out of the contact area, the tread regains its original length and the peripheral velocity returns to \( \Omega \cdot R \) with \( R \) the unloaded radius. As a consequence, the spin speed of the wheel with a pneumatic tyre under conditions of free rolling is less than that of a rigid wheel and:

\[ R_i < R_e < R \]

It means that the centre of rotation of the wheel usually lies somewhere below the surface. The effective rolling tyre under free rolling also behaves different with varying tyre load compared to the loaded tyre radius. A loaded radius behaves almost linear in the tyre load \( F_z \), i.e. the tyre behaves as a linear spring in vertical direction. The effective rolling radius varies significantly with tyre load. This can be described based on empirical fit as follows (see [1]):

\[
R_{e,\text{free-rolling}} = R - \rho_0 \cdot [D \cdot \arctan(B \cdot \frac{\rho}{\rho_0}) + E \cdot \frac{\rho}{\rho_0}]
\]

with tyre deflection \( \rho \), tyre deflection \( \rho_0 \) for nominal tyre load \( F_{zo} \), and fitparameters \( B \), \( D \), \( E \) which may vary according to:

\[ 3 < B < 12 \] : B stretches the effective tyre characteristic curve along the \( F_z \) -axis (ordinate). B large means a large slope at \( F_z = 0 \).
\[ 0.2 < D < 0.4 \] : shift from asymptote at high wheel loads
\[ 0.03 < E < 0.25 \] : with low values of E for stiff tyres

![Figure 4.: Effective and loaded tyre radius under conditions of free rolling](image_url)
An example of the variation of $R_l$ and $R_e$ is shown in figure 6 for $B = 10$, $D = 0.25$ and $E = 0.05$. The tyre stiffness is taken as $2 \times 10^5$ N/m. The unloaded radius $R$ is taken as 0.32 m and we choose $F_{z0} = 4000$ N. We have also varied the parameters to illustrate the range of possible effective rolling radius characteristics.

The effective rolling radius turns out to increase with increasing speed and increasing inflation pressure. The variation with speed is strongly dependent on the tyre carcass structure.

A radial-ply tyre rolling radius appears to be almost constant for varying speed in contrast with the diagonal-ply (bias-ply) tyre. This phenomenon has to do with the radial response of the tyre to higher circumferential speeds.

2. The rolling tyre.

Let us discuss the rolling tyre in more detail, see figure 7 (see also [20]). With the tread entering and moving through the contact area, the distance to the wheel centre changes from the unloaded radius to the loaded radius and back to the unloaded radius. With the peripheral speed in the contact area corresponding to the effective rolling radius in between these values, points in the contact area need to catch up with this peripheral speed at the both ends of the contact area where the distance of contact points to the wheel centre exceeds $R_e$. As a consequence, one observes rearward slip at these parts. With a similar argument, the points of the tyre circumference are slowed down in speed in the centre part of the contact area, corresponding to forward slip. Integration of the slip over contact area results in the global performance related to the shear stress as indicated in figure 7. The peripheral speed with respect to the wheel centre is shown in the lowest graph in figure 7, reducing from the unloaded speed $\Omega R$ just before the contact area to the speed $\Omega R_e$ within the contact area.

The total longitudinal net force, determined from integrating the shear stress over the contact area will be a nonzero, negative force, known as the rolling resistance force. This rolling resistance force corresponds to a moment acting around the wheel centre, being balanced by the moment resulting from the tyre load. Consequently, the net tyre load will have to act along a force line, slightly in front of the wheel centre.
3. The tyre under braking or driving conditions.

Now consider a tyre under a braking torque, as indicated in figure 8. The brake torque \( M_y \) has to be balanced by moments due to a brake force \(-F_x\) and the tyre load \( F_z\).

The offset of the tyre load in front of the wheel centre increases with respect to the free rolling tyre. The tyre will experience a slip speed of wheel w.r.t. ground, reducing the angular speed and therefore increasing the effective rolling radius \( R_e \). If \( M_y \) is large enough, \( R_e,\text{braking} \) will exceed the loaded radius.

The total longitudinal shear stress in the contact area now consists of a part due to free rolling (dashed in figure 8) and a superimposed shear stress caused by braking. As a results, the major part of the tyre in the contact area is stretched due to the braking torque. Tread elements entering the contact area first try to adhere to the road surface, with the longitudinal deflection and therefore the shear stress increasing linearly along the contact zone. At a certain point, the shear stress reaches the limits of friction (\( \mu \sigma_z \) with local road friction \( \mu \) and normal stress \( \sigma_z \) under Coulomb law) and the treads start to slide. As a result, the shear stress drops down along the rear part of the contact zone. In a similar way as discussed for a free rolling tyre, one arrives at a distribution of the peripheral velocity of treads (w.r.t. the wheel center) as shown in the bottom part of figure 8.

Note that, in general, sliding starts at the rear of the contact area and extends towards the front part of the contact area for increasing brake torque, until finally sliding is apparent along the full contact area.

In case of a tyre under driving conditions, the angular speed is increased and therefore the effective rolling radius \( R_e,\text{driving} \) decreased. The drive torque has to balance moments resulting from a driving force in the contact area and the tyre load. The offset of the tyre load line in front of the wheel centre is decreased with respect to the case of the free rolling tyre. The shear stress is now built up from the free rolling distribution plus a triangular shaped pattern along the contact area, and the tyre tread material is experiencing a compression.

3.1. Practical brakeslip

We introduce the practical longitudinal brakeslip \( \kappa \) as follows:

\[
s_x \equiv -\kappa = \frac{V_x}{V} = \frac{V_x - \Omega R_e}{V_x} \equiv -\frac{\Omega - \Omega_0}{\Omega_0}
\]
with slip speed $V_{sx}$ of tread elements with respect to the road surface (obtained from the difference of the forward tyre speed $V_x$ at the wheel centre with respect to the road surface, and the peripheral speed $\Omega.R_e$ of tread elements with respect to the wheel centre), and the angular speed $\Omega_0$ under free rolling conditions. Observe that, under braking, $\kappa$ varies between -1 (locked wheel, $\Omega = 0$) and 0 ($V_{sx} = 0$).

When a driver starts braking, the angular speed per wheel is changed, where the rotational wheel inertia $I_{\text{wheel}}$ is decelerated by the resultant of the brake torque and the tyre brake force:

$$I_{\text{wheel}} \cdot \dot{\Omega} = -M_x - R_I \cdot F_x(\kappa)$$

with $F_x > 0$ in positive x-direction (i.e. $F_x < 0$ in case of braking). This equation is part of a larger set of equations to solve the braking problem for a vehicle. Clearly, the forward vehicle speed (being included in the above angular wheel velocity equation through the slip $\kappa$) will decrease. The resulting forward vehicle speed follows from another equation describing the balance of the vehicle inertia deceleration and the wheel forces:

$$m_{\text{vehicle}} \cdot \dot{V}_x = \sum_{\text{all wheels}} F_x(\kappa)$$

### 3.2. Longitudinal slip characteristics.

In order to solve the angular wheel velocity equations for each wheel (with possibly all different slip values), one requires a description of $F_x$ in terms of practical slip $\kappa$. A typical behaviour of this longitudinal characteristic tyre behaviour is shown in figure 9. In the left-hand picture, we have plotted $-F_x$ (with brake force $F_x$) versus $|\kappa|$ whereas in the right-hand picture, we have plotted $-\mu_x \equiv -F_x/F_z$, the so-called normalized tyre force.

*Figure 9: Brake force vs. longitudinal slip*
(also known as the longitudinal force coefficient), for various values of the tyre load. Usually, the curves will not exactly pass the origin (due to rolling resistance, inaccuracies in the tyre). Clearly, the longitudinal tyre force is close to being proportional to the tyre load but not quite. The longitudinal slip stiffness, being the slope of the curve for $F_x$ at $\kappa = 0$, tends to decrease more then proportional with $F_z$ for increasing tyre load. One observes a peak value and a saturation value in both pictures, for the longitudinal force coefficient indicated as $-\mu_{xp}$ (peak value) and $-\mu_{xs}$ (the limit of $-\mu_x$ for pure sliding, i.e. at $\kappa = -1$). The peak value is obtained for brakeslip around 0.1 and 0.15 in absolute value ($10–15\%$ slip). For small brakeslip, this characteristic can be approximated by a linear relationship, with slope being the longitudinal slip stiffness.

The peak value is the optimal value to brake, but just beyond the slip corresponding to this optimal value, the wheel will lock in very short time. That is the reason why nowadays almost all vehicles are equipped with anti-lock systems, in order to prevent to excessive brake slip. In the same way, one may discuss driveslip, and the risk of spinning of the wheel in case of too high traction. This phenomenon can be prevented using traction control systems.

### 3.3. Road conditions and brakeslip.

The normalized tyre force $-\mu_x$ and (therefore also the longitudinal tyre force itself) depends essentially on the tyre-road conditions, that means on things like:

- road roughness. Pavement exhibits three types of roughness, micro-texture (with wavelength less than 0.5 mm), macro-texture (wavelength between 0.5 mm and 50 mm) and mega-texture (wavelength exceeding 50 mm), see [33]
- tyre tread wear
- wet conditions (wet, possible hydroplaning, snow, ice,…)

Micro- and macro-texture are schematically shown in figure 10. Macro-texture is related to the overall roughness of the road resulting from the number, type and size of stone chippings, whereas micro texture has to do with the roughness of the individual chippings. Idealized texture leads to sufficient drainage and significant hysteretic friction (local pressures) at the cost of tyre wear. Tips should preferable be sharp to have good friction even under wet conditions, but that leads to abrasive wear. The existence of micro-texture is due to the typical asphalt ingredients (silica, sand, quartzites).

Macro-texture and micro-texture vary in time. It is known from drain asphalt that, due to the situation of many small contact zones between rubber and ground, there is more
polishing effect and therefore rounded asperities, with impact on the adhesive properties of the tyre road contact. Roughly speaking one might say that macro-texture is related to a strong velocity dependence of the tyre-road contact under wet/rain conditions, whereas micro-texture is related to the slightly wet or dry-adhesive aspects. See also [17].

3.3.1. Wet road conditions.
Under wet road conditions, the longitudinal force coefficient maximum level drops, to levels in the order of 0.6 - 0.8 for a wet road, to 0.4 - 0.5 for snow, and to levels of 0.2 - 0.4 for ice.

![Figure 11.: Contact area on a wet road for different speeds](image)

A special case is given when water is present on the road. In order to maintain contact between tyre and road, the water has to be evacuated, and this property may be improved by adjusting the tread block pattern of the tyre (longitudinal grooves, or grooves curved in an outward direction guiding the water in a radial direction away from the tyre). With increasing speed, there is less time to remove the water and the contact zone is further reduced, see figure 11 for an example [4] for three different speeds.

![Figure 12.: The effect of road conditions and speed in the longitudinal force coefficient](image)

At a certain speed, the tyre may float entirely on a film of water (hydroplaning), and the friction coefficient drops to very low values (< 0.1). In other words, hydroplaning
occurs when a tyre is lifted from the road by a layer of water being trapped in front of and under a tire. One usually distinguishes between **dynamic hydroplaning** (water is not removed fast enough to prevent loss of contact) and **viscous aquaplaning** when the road is contaminated with dirt, oil, grease, rubber-parts, leaves etc. Usually, regular rain will wash this away, but especially after a long dry period with dirt, dust etc. having piled up, a sudden rain may result in a more viscous mixture on the road causing unexpected dangerous (i.e. low friction) conditions.

In figure 12, the qualitative effect of road conditions and velocity on the longitudinal force coefficient is shown. These graphs agree with results, presented in [10]. One observes a minimal effect of velocity in case of a dry road in contrast to the situation when the road is wet. In the latter case, the brake force drops significantly with vehicle velocity.

### 3.3.2.: Road conditions, wear, tyre load and speed

The impact of aquaplaning in combination with wear is illustrated in figure 13, taken from [10], with the locked wheel longitudinal force coefficient value plotted against vehicle velocity. As expected, this locked wheel value is further reduced under tread wear conditions. The hydroplaning velocity is reduced with increasing water layer depth. Similar results for the peak wheel longitudinal force coefficient are shown in figure 14 (from [14]).

The combined effect of speed, road condition and tyre load is shown in figure 15, in terms of the peak longitudinal coefficient $\mu_{xp}$ and the sliding longitudinal coefficient $\mu_{xs}$, from [5].

Sliding coefficients are more sensitive to speed than the peak values. The sensitivity of speed on the peak value increases if the road gets wet.

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**Figure 14.: Maximum friction coefficient for different water film heights and vehicle speeds [14]**
There are different ways to describe the longitudinal slip behaviour using tyre models. One distinguishes between physical models and empirical models. A physical model describes the tyre on the basis of the recognized physical phenomena during braking, usually in a simplified way. Such simplified models do not aim to give a quantitative description of the tyre handling performance, but merely give an explanation of the qualitative phenomena (shape of the curve, trends in the sense that the impact of changing vehicle speed, road conditions etc. are well covered, etc.). These models can be used for longitudinal behaviour, cornering behaviour and combined slip behaviour, and will therefore be addressed in section 5.

More complex physical models are for example Finite Element models, applied in order to derive quantitatively correct tyre performance based on a detailed description of the tyre structure and material properties. That means that FE models form a link between tyre design and tyre performance. However, FE-models are very time consuming, both in CPU-time and in preparation time (setting up the model).

Empirical tyre models are based on a similarity approach where experimental results are used to find parameters to tune a certain mathematical description. A well-know empirical tyre model is de Magic Formula model, due to H.B. Pacejka, therefore also often referred to as the Pacejka model. This Pacejka model has been implemented in many different versions. We refer here to the version being implemented in ADAMS/tyre, originated from DELFT-TYRE, see [22] and [1]. We note here that, different from the preceding analysis, the Pacejka tyre model assumes a z-axis pointing upward. i.e. with the y-direction pointing port side.

### 3.5. The pure slip longitudinal Magic Formula description

The basic mathematical formula describing the longitudinal characteristics is given by the so-called sine-version, given by:

\[ Y(x) = F_x(x) = D \sin(C \arctan(B.x - E.(B.x - \arctan(B.x)))) + S_y \]
with $Y(x)$ being either $F_x$ or $F_y$, and $x - S_H$ being either the longitudinal slip $\kappa$ or the lateral slip $\tan(\alpha)$ for slip angle $\alpha$ (see next section). The parameters $S_H$ and $S_V$ are so-called shifts to allow the curve not to pass through the origin (i.e. $Y(x) = 0$ does not automatically imply $x = 0$).

$D$ is related to the peak of the longitudinal force coefficient and the wheel load:

$$D = \mu_x F_z$$

Neglecting camber, the Magic Formula give for $\mu_x$:

$$\mu_x = (p_{sd1} + p_{sd2} df_z) \text{ with } df_z = \frac{F_z - F_{z0}}{F_{z0}}$$

with nominal tyre load $F_{z0}$.

The nominal tyre load is related to the maximum admissible static load for the specific temperature and speed index, usually referred to as the ETRTO value (European Tyre and Rim Technical Organisation). The speed index indicates the maximum speed for which the tyre is allowed to be used, before it destroys itself due to overheating, as a result of high-frequency standing waves responsible for a strong increase of internal deformation power being converted into heat.

Choosing the nominal value $F_{z0}$ being equal to 80% of this ETRTO value, a reasonable choice for $F_{z0}$ is listed in table 1.

<table>
<thead>
<tr>
<th>Class</th>
<th>$F_{z0}$ [N]</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compactclass</td>
<td>3000</td>
<td>VW-Polo</td>
</tr>
<tr>
<td>Middle class</td>
<td>5000</td>
<td>VW-Passat, BMW-5,..</td>
</tr>
<tr>
<td>Topclass</td>
<td>6000</td>
<td>Audi A8</td>
</tr>
</tbody>
</table>

*Table 1.: Some typical values for the nominal tyre load $F_{z0}$.*

Hence, a specific nominal tyre load is related to a class of tyres, with the same maximum allowable operating speed. Different nominal tyre loads refer therefore to different classes of tyres, in contrast to the variation in tyre load for one specific tyre (due to static load variations, load transfer during cornering, etc.).

Other parameters in the Pacejka tyre formula for pure longitudinal slip can be expressed as follows (neglecting camber):

$$BCD = F_z \cdot (p_{Kx1} + p_{Kx2} df_z) \cdot \exp(p_{Kx3} \cdot df_z)$$

$$E = (p_{Ex1} + p_{Ex2} df_z + p_{Ex3} df_z^2) \cdot (1 - p_{Ex4} \cdot \text{sign}(\kappa))$$

$$S_H = (p_{Hz1} + p_{Hz2} df_z)$$

$$S_V = F_z \cdot (p_{Kx1} + p_{Kx2} df_z)$$
A typical value for $C = C_x$ is given by $C_x = 1.68$. For further suggestions for parameters, we refer to [1].

4. The tyre under cornering conditions

Let us consider a tyre under cornering conditions, as indicated in figure 16. Under cornering conditions, there exists a local velocity vector, being in general not parallel to the wheel centre plane. This wheel centre plane is defined as the symmetry plane of the tyre such that forces acting in the symmetry plane do not contribute to the lateral force for the tyre.

In the front part of the contact area, the treads of the tyre try to follow this local speed direction, resulting in a displacement along the tyre circumference within the contact area, increasing linearly from zero (just in front of the contact area) up to a situation where the induced lateral shear stress just reaches the maximum possible shear stress level, i.e. $\mu \sigma_z$ with local road friction $\mu$ and normal stress $\sigma_z$ under Coulomb law. We have discussed a similar phenomena for braking and driving (traction) of the tyre. Beyond that point, the treads of the tyre will slide leading to a reduction of the shear stress in the direction of the contact area rear end. Clearly, when sliding and in the absence of longitudinal slip, the lateral shear stress will be equal to $\mu \sigma_z$. With $\sigma_z$ reducing to zero at the edges of the contact area, the friction limits for the shear stress will decrease further, and sliding is likely to extend until the contact area rear end.

Deflection of the tyre is due to two separate effects, (1) the deflection of the contact rubber, i.e. of the treads, and (2) deflection of the belt.

Both compliances allow the tyre to direct itself to the local speed direction, but the stiffness are different. In terms of physical models, one may distinguish here between the so-called brush model and the stressed string model. Both will be treated in more detail in section 5.

We introduce the practical lateral slip as $-\tan(\alpha)$, i.e.
\[ s_y = -\tan(\alpha) = \frac{V_{sy}}{V_x} = \frac{V_y}{V_x} \]

with slip speed \( V_{sy} \). As we will see later, the practical slip quantities correspond with a description of tyre deflection in terms of deformed quantities. An alternative approach might be to express slip in terms of the undeformed coordinate system. This will result in the so-called theoretical slip quantities, defined as:

\[ \rho_x = \frac{V_{sx}}{V_r} ; \quad \rho_y = \frac{V_{sy}}{V_r} \]

Using

\[ \kappa = \frac{V_r - V_x}{V_x} = \frac{\Omega R - V_x}{V_x} \]

one easily arrives at the following relationship between practical and theoretical slip quantities:

\[ \rho_x = \frac{s_x}{1 - s_x} ; \quad \rho_y = \frac{s_y}{1 - s_x} \]

As we observed before, the practical brake slip \( s_x \) varies between 0 and 1, whereas under driving conditions, \(-\infty < s_x < 0\), i.e. the practical driveslip may attain very large absolute values in case of wheel spinning on the spot. In contrast to the practical slip, the theoretical longitudinal slip remains bounded under driving conditions but may grow to large absolute values in case of braking when the wheel gets locked.

**4.1. Vehicle cornering performance**

Vehicle dynamics analysis includes relationships between slip angles at front and rear axles (and possibly at the separate wheels) and global vehicle performance output variables such as yaw-rate and lateral vehicle speed (or, equivalently, the body side slip angle). See figure 17 for a schematic layout of a vehicle under cornering conditions. The four tyre forces balance the centripetal force, acting on the vehicle in local lateral direction:

*Figure 17.: Vehicle handling, schematically*
\[ m_{\text{vehicle}} \dot{v}_y + V_{\text{vehicle}} \dot{r} = \sum_{\text{all wheels}} F_y(\alpha) \]

with lateral vehicle speed at the centre of gravity, \( v_y \) and yaw rate \( r \). In addition, the moments of the four tyre around the centre of gravity have to balance the total inertial moment, approximated by:

\[ \sum_{\text{front wheels}} a.F_y(\alpha) - \sum_{\text{rear wheels}} b.F_y(\alpha) \]

with vehicle moment of inertia in vertical z-direction \( I_{\text{vehicle}} \). Note that, for small slip angles and steering angle, the vehicle speed can be approximated by \( V_x \) (and vice versa).

Usually, one assumes the slip angles to be identical for both front wheels, and likewise for both rear wheels. Slip angles are defined by the orientation of the local velocity vector, relative to the wheel symmetry plane. With the variables as indicated in figure 17, one easily finds for the local outward lateral velocity at the front and rear wheels:

Local speed front tyre : \(-V_y - r.a\)

Local speed rear tyre : \(-V_y + r.b\)

Hence,

\[ \alpha_1 \approx \tan(\alpha_1) = \frac{V_y + r.a}{V_{\text{vehicle}}}; \quad \alpha_2 \approx \tan(\alpha_2) = \frac{V_y - r.b}{V_{\text{vehicle}}}; \]

4.2. Lateral slip characteristics

In order to solve the two equations above, one requires a description of the lateral force in terms of practical slip \( \tan(\alpha) \approx (\alpha) \). A typical behaviour of this lateral characteristic tyre behaviour is shown in figure 18.

![Figure 18.: Cornering force vs. slipangle, camber angle = 0](image-url)
Similar as in case of braking or driving, we have plotted both $F_y$ and $\mu_y = F_y/F_z$, the so-called normalized tyre force or the lateral force coefficient (sideforce coefficient), for various values of the tyre load. Again, one observes the tyre force to be close to being proportional to the tyre load but not quite. One observes peak values and saturation values in both pictures, indicated for the lateral force coefficient as the peak value $\mu_{yp}$, and $\mu_{ys}$ as the limit of $\mu_y$ when the tyre is drifting for large slip angle. For small slipangle, this characteristic can be approximated by a linear relationship, with slope being the normalized lateral slip stiffness or the normalized cornering stiffness.

4.3. Side force coefficient for different textures and speeds
Some values of the sliding sideforce coefficient $\mu_{ys}$ for different texture depth are shown in figure 19, under wetted conditions. One observes an increase in side friction force with texture depth (except for the Bridport surface). The Bridport surface is rather smooth (pebbles included), eliminating the adhesion coefficient of friction for wetted sliding conditions. Observe also the effect of speed, with increased speed lowering the friction, especially with small texture depth (as expected).

![Figure 19: Sideforce coefficient ($\mu_{ys}$) for different texture depths and velocities](image)

4.4. Cornering stiffness versus tyre load
The lateral slip stiffness or cornering stiffness $C_{ye}$ being the slope of $F_y(\alpha)$ at $\alpha = 0$ (the slope in the left-hand picture in figure 18, see section 4.2), tends to decrease more then proportional with $F_z$ for increasing tyre load. The cornering force is shown in figure 20 vs. tyre load. This non-linear relationship is important in the sense that, during cornering, the tyre load of the outer wheel will
increase whereas the inner wheel load will decrease. Due to the nonlinear dependence of cornering stiffness on tyre load, the change in cornering stiffness at the outer wheel is exceeded in absolute value by the change at the inner wheel. For this reason, the average cornering stiffness for the full axle is decreased. With different roll stiffnesses at front and rear axle, this works out differently at both axles. We will see later that the cornering performance of the vehicle strongly depends on the axle characteristics. As a result, this performance will change with increasing roll. Hence, by actively controlling the roll stiffness at front and/or rear axle, one is able to improve the vehicle handling performance.

![Figure 20: Cornering stiffness vs. tyre load](image)

*Figure 20.: Cornering stiffness vs. tyre load*

In figure 21, ranges of typical values of the cornering stiffness coefficient (cornering stiffness, divided by the tyre load) are shown vs. tyre load for passenger car and truck

![Figure 21: Cornering stiffness coefficient (from normalized tyre force) for passenger car and truck tyres, from [2]](image)
tyres. Truck tyres experience quite a large tyre load variation, compared to passenger car tyres. Clearly, these tyres have to be designed with minimum impact of load on tyre performance. This is illustrated in figure 21.

4.5. Pneumatic trail and aligning torque

Figure 16 indicates that the sideforce acts at a small distance behind the wheel centre. This distance is called the pneumatic trail. At small slip (small $\alpha$), there is almost no sliding and the adhesion part extends almost over the entire contact area. This corresponds to a situation where the shear stress profile is very unsymmetrical along the contact area, with a rather large pneumatic trail. With slip increasing, the sliding area increases towards the front end of the contact area. Under Coulombs law, the shear stress in the sliding area follows $\mu \cdot \sigma_z$. The normal contact stress $\sigma_z$ throughout the contact area is shown in figure 22 for both a radial-ply tyre and a bias-ply tyre.

Both pictures in figure 22 confirm that the resultant vertical contact force acts slightly in front of the wheel centre (as discussed before), meaning that the pneumatic trail may even become negative for excessive sliding. Observe also the different behaviour for bias-ply and radial-ply tyres at the shoulders of the tyres. We have seen similar concentrations in shear stress in the previous sections.

![Figure 22.: Normal contact stress profile for different tyres, from [11]](image)

Hence, we have a sideforce $F_y(\alpha)$, starting at small values at $\alpha = 0$ and growing to a maximum value ($\mu \cdot F_z$) whereas the pneumatic trail $t_p(\alpha)$ starts at large values, reducing to small values with even negative values for excessive slip. Pneumatic trail times sideforce yields the so-called aligning torque $M_z$. This torque is called aligning since it aims to orient the tyre in the speed direction. It works against the lateral deformation due to the lateral force. With:
for residual torque $M_{zr}$ (small torque resulting from inaccuracies in the tyre design, rapidly decreasing in absolute value with increasing slip angle) we expect this aligning torque to start close to zero for $\alpha = 0$, to grow in absolute value but to decrease again with the pneumatic trail for increasing slip. We have plotted the pneumatic trail and the aligning torque in figure 23.

Comparing figure 23 with figure 18 (see section 4.2), we see that the aligning torque passes its maximum at a slip angle value, smaller than at the maximum of the sideforce where the tyre starts sliding. The torque from the combined effect of mechanical trail (castor) and pneumatic trail is felt by the driver through the steering wheel. Reduction of the aligning torque in absolute value should warn the driver that he or she is approaching a situation with increased risk of skidding of the front axle due to excessive understeer.

### 4.6. The empirical Magic Formula

Referring to the empirical Magic Formula sine version describing the lateral characteristics:

$$F_y(x) = D \sin(C \arctan(B \alpha - E(B_x - \arctan(B x \alpha)))) + S_v$$

the different coefficients $B$, $D$ and $E$ can be expressed as follows (neglecting camber):

$$D = \mu_{yp} F_z$$

with:

$$\mu_{yp} = (p_{yd1} + p_{yd2} df_z) \quad \text{with} \quad df_z = \frac{F_z - F_{z0}}{F_{z0}}$$
with nominal tyre load $F_{z0}$. Furthermore

$$BCD = p_{Ky1} \cdot F_{z0} \cdot \sin(2 \cdot \arctan(\frac{F_z}{p_{Ky2} \cdot F_{z0}}))$$

$$E = p_{Ey1} + p_{Ey2} \cdot df_z$$

$$S_H = (p_{Hy1} + p_{Hy2} \cdot df_z)$$

$$S_V = F_z \cdot (p_{Ky1} + p_{Ky2} \cdot df_z)$$

### 4.7. Camber

So far, we neglected camber. The **camber angle** is defined as the angle between the wheel plane and the normal of the road in the transverse plane of the vehicle, see figure 24. The presence of a camber angle $\gamma$ produces a lateral force, which is usually much smaller than the side force due to sideslip $\alpha$. This can be explained as follows. A wheel under a camber angle would move over a circular track. The direction of motion of the wheel is forced by the vehicle velocity vector. For example, the wheel may be going straight ahead. As a result, local shear stresses arise in the contact area, building up a camber force.

For a motorcycle, the camber force is the main force between tyre and road, that prevents the tyre to slide.

In the linear range, the side force can be expressed in terms of slip angle and camber angle in the following way:

$$F_y(\alpha) = C_{y\alpha} \cdot \alpha + C_{yy} \cdot \gamma$$  ; small $\alpha$ and $\gamma$

with cornering stiffness $C_{y\alpha}$ and **camber stiffness** $C_{yy}$, defined as:

$$C_{y\alpha} = \frac{\partial F_y}{\partial \alpha}(\alpha = 0, \gamma = 0)$$

$$C_{yy} = \frac{\partial F_y}{\partial \gamma}(\alpha = 0, \gamma = 0)$$
For different light truck tyres, for various loads and tyre inner pressure, the camber thrust coefficient (ratio of camber stiffness and tyre load) is shown in figure 25. One observes the low value in the order of 0.01 until 0.03, to be compared to values between 0.5 and 1.0 for the normalized cornering stiffness.

In the nonlinear range, the above empirical formulas have to be corrected by including the dependency on camber. We refer to [1], [22] and [23] for further details. We have varied the camber angle and calculated the side force, based on the Magic Formula parameters, presented in [1]. The result is shown in figure 26. The camber angle is changed such that the side force is decreased with increase of camber angle in

![Graph showing lateral force vs. slip angle for varying camber angle](image)

*Figure 26.: Side force vs. slip angle for changing camber angle (in rad)*

![Graph showing pneumatic trail & aligning torque for varying camber angle](image)

*Figure 27.: Pneumatic trail & aligning torque for varying camber angle*
absolute value. This is the usual case, with the carbody rolling outward, leading to a reduction of the side force. The corresponding pneumatic trail and aligning torque for varying camber angle are shown in figure 27.

4.8. The Gough plot
An interesting way of presenting tyre characteristics in a graphical way, is given by the so-called Gough-plot where side force $F_y$ is plotted against $M_z$, neglecting shifts, see figure 28.

This plot is very illustrative, since it shows the dependency of the tyre characteristics on slip angle, tyre load and pneumatic trail in one picture. It clearly identifies the different impact of slip-angle (dashed) and the tyre load (solid). Lines of constant pneumatic trail are straight lines, distributed purely radial. For larger tyre load, the lateral force increases. The aligning torque increases as well, but it starts to decrease in slip angle, when the lateral force is still increasing in the slip angle. For larger slip angle, also the side force starts to saturate.

This plot shows that already for small side force and aligning torque (i.e. normal non-extreme cornering), a clear distinction can be made between the impact of $\alpha$ and $F_z$, the latter of which is close to the impact of road friction. Consequently, this plot suggests itself as a way of monitoring side slip and road friction from the tyre performance, a fact that has been exploited successfully by Pasterkamp [28].
5. Combined braking and cornering

The discussion in the preceding section deals with pure slip, i.e. in cases where the car is either cornering, or braking/driving. When a driver torque or brake torque is applied during cornering, the total horizontal force is acting not in the longitudinal or lateral direction, and the cornering force is reduced. Likewise, applying a side force while braking or driving will reduce the longitudinal force, i.e. the braking or driving potential of the tyre. With the total force:

\[ F_{\text{tyre}} = \sqrt{F_x^2 + F_y^2} \]

we can define the resultant force coefficient as

\[ \mu_{\text{tyre}} = \frac{F_{\text{tyre}}}{F_c} \]

We have plotted the longitudinal and lateral force versus longitudinal slip in figure 29, for varying slip angle. One observes a decrease of \( F_x \) and increase of \( F_y \) for increasing slip angle.

For small braking or driving, the sideforce is dominant. For large braking or driving, there is hardly any potential left for the sideforce, and the sideforce appears to be small compared to values for small longitudinal slip.

5.1. Polar diagrams, \( F_x \) vs. \( F_y \) and \( F_x \) vs. \( M_z \)

In figure 30, we have included so-called polar plots, with \( F_x \) plotted against sideforce \( F_y \)

\[ \alpha = -0.1 \ -0.05 \ 0.0 \ 0.05 \ 0.1 \ 0.15 \ 0.25 \ 0.4 \]

Figure 29.: Interaction between longitudinal forces and side forces

Figure 30.: Polar diagrams, \( F_x \) vs. \( F_y \) and \( F_x \) vs. \( M_z \) for constant slip angle
and against aligning torque $M_z$, respectively. These diagrams are nonsymmetrical in $F_x$, which is due to the carcass-stiffness. The longitudinal force, acting in the contact zone in the direction of the local rotated longitudinal coordinate axis, contributes to both the total lateral force $F_y$ and to the aligning torque $M_z$. Clearly, this works out just opposite when the side force changes sign.

Observe in figure 30 that the $F_x$-$F_y$ diagram is close to a circular area. One would expect the saturation of the total horizontal force $F_{tyre}$ to occur when $F_{tyre} = \mu F_z$ with road friction $\mu$. This would exactly lead to a circle, describing the maximum possible values for $F_{tyre}$.

### 5.2. The Magic Formula for combined slip.

The magic Formula describes combined slip using weighting functions for the pure slip characteristics:

$$F_x(\alpha, \kappa) = G_{x\alpha}(\alpha, \kappa) F_{x,\text{pure}}(\kappa)$$
$$F_y(\alpha, \kappa) = G_{y\alpha}(\alpha, \kappa) F_{y,\text{pure}}(\alpha) + S_{y\kappa}$$
$$M_z(\alpha, \kappa) = -t_p(\alpha_{t,eq})(F_x(\alpha, \kappa) - S_{v\kappa}) + M_{z\kappa}(\alpha_{r,eq}) + s F(\alpha, \kappa)$$

for equivalent slip angles $\alpha_{t,eq}$ and $\alpha_{r,eq}$ (depending on longitudinal slip), residual torque $M_{z\kappa}$ and moment arm $s$ of $F_x$ contributing to $M_z$. See [23] for more details. Again we observe a contribution to the aligning torque from the longitudinal force, due to the carcass flexibility.

The weighting functions in the above expressions can be described by cosine versions of the Magic Formula, with Magic Formula parameters tuned from experiments. It can be shown (see for example [23] and [10]) that the combined slip forces can be approximated well by:

$$F_x(\alpha, \kappa) = \frac{\rho_x}{\rho} F_{x,\text{pure}}(\rho)$$
$$F_y(\alpha, \kappa) = \frac{\rho_y}{\rho} F_{y,\text{pure}}(\rho) \quad \rho = \sqrt{\rho_x^2 + \rho_y^2}$$

for theoretical slip values $\rho_x$ and $\rho_y$, introduced earlier. A similar successful approximation can be derived from the practical slip quantities:

$$F_x(\alpha, \kappa) = \frac{\kappa}{s} F_{x,\text{pure}}(s)$$
$$F_y(\alpha, \kappa) = \frac{\tan(\alpha)}{s} F_{y,\text{pure}}(s) \quad s = \sqrt{\kappa^2 + \tan^2(\alpha)}$$

We refer to [29] where these approximations have been studied in detail.
Some results are shown in figure 31 where the polar plot $F_x$ vs. $F_y$ and both tyre forces vs. longitudinal slip for approximations cf. Magic Formula and cf. the above similarity approach are shown. We observe the unrealistic symmetry in the approximation. On the other hand, both type of curves are not far apart, and one should realize that the approximated curves are based on the pure slip characteristics, i.e. they do not require combined slip measurements. And if more accurate results are needed, the approximated curves give a very good first estimate, to verify the test results. Note that, in many analyses, the high accuracy from the Magic Formula empirical approach are not required, and the approximated values may serve as a good alternative. Pure slip characteristics are often easily estimated from published graphs, i.e. even pure slip measurements may not be necessary to find a satisfactory description of the pure slip characteristics, and through that, a combined slip description.

5.3. Physical tyre models, requirements
We have mentioned earlier two possible approaches to derive physical tyre models:

1. the brush model
2. the stressed string model

These two important physical models are schematically shown in figure 32. These approaches are two special examples of more general physical models, which will be discussed here in some more detail with special emphasis to brush models. The models all give a general description of the tyre under full combined slip conditions. Therefore, these models will be addressed in this section. Note however, that the models can easily be simplified to pure slip in either lateral (i.e. cornering) or longitudinal (i.e. braking or driving) direction.

Physical models should account for:

- frictional properties in the tyre-road interface
- distribution of the normal contact force
- stiffness of the tread rubber
- stiffness of the carcass.

Models of the carcass commonly encountered in the tyre literature can either be a spring, they can be of beam type or of stretched string type. The exact representation of the carcass by a beam instead of a stretched string is more difficult because of the fact that the differential equation for the shape of the deformed peripheral line of the carcass becomes of fourth instead of the second order. For the study of steady state tyre behaviour, most authors approximate the more or less exact expressions for the lateral

As an extension of the model of Fromm (brush approach) and of Julien (see [15] for further references) who did not consider carcass elasticity, Fiala [6] and Freudenstein [9] developed theories in which the carcass deformation has been approximated with a symmetric parabola determined only by the lateral force. Böhm [3] and Borgmann [4], the latter without tread elements, use asymmetric approximate shapes determined by both the lateral force and the aligning torque. In [24] and [25], Pacejka describes the steady-state tyre characteristics for a stretched-string tyre model with and without tread elements attached to the string. The lateral stiffness distributed as measured on a slowly rolling tyre in terms of influence of Green’s functions (cf. [31]) may be employed in a model for the slipping tyre as has been discussed in [26] and [27]. The combination of stressed string model and brush-model under arbitrary combined slip conditions has been considered by [30]

5.4. Performance of different physical tyre models
Frank [7] has carried out a thorough comparative investigation of the various one-dimensional models. He employed a general fourth-order differential equation with which stretched string, beam and stretched beam tyre models can be examined. He
obtained the exact solution of the stationary side slip problem (no longitudinal slip included), and comparison with the various tyre models revealed that the stretched string type of model was more suitable for the simulation of bias-ply tyres, whereas the beam model (i.e. with belt-bending taken into account) was more appropriate for the radial-ply tyre.

The following models were compared:

a. stretched beam model. The belt is taken as a beam under tension, i.e. with bending stiffness taken into account.
b. beam model. Similar to model a, however with the tension force neglected.
c. approximate solution for the lateral force by Fiala, see [6].
d. model of Fromm, taking only tread deformation into account with the carcass assumed to be rigid.

Before we discuss the results, we first give some understanding about belt models. Consider the tyre top-view shown in figure 33.

Distinction is made between belt deflection and tread deflection. The belt deflection can, in general, be described by a stretched beam. That means that the steady state lateral displacement $y(x)$ of the belt, in terms of the position $x$ along the wheel centre plane, satisfies a fourth order differential equation:

$$EI \frac{d^4y}{dx^4} - S \frac{d^2y}{dx^2} + K.y = q_y(x)$$

for bending stiffness $EI$, tension force $S$, carcass stiffness per unit length $K$ and lateral side force per unit length $q_y(x)$. With $S = 0$, the beam is non-stretched. For $EI = 0$, the equation reduces to the stretched string equation. The Fiala approximation describes the lateral force $F_y$ as a third order expression in the slip angle $\alpha$. The model of Fromm neglects the carcass deflection, i.e. only tread deflection is described leading to the brush model.

Figures 34 and 35 present the calculated characteristics (taken from [9]) of models a – d.

![Figure 33.: Lateral tyre deformation](image)
The parameters in cases a – c were chosen in such a way as to give a best fit to experimental data for the cornering force at small slip angles. The curves d show the result when carcass elasticity has been neglected. The coefficient of friction $\mu$ was taken constant and the vertical pressure distribution was taken from measurements, lying between a parabolic and an elliptic shape. The positive aligning torque at high slip values arose due to a slightly asymmetric shape of the pressure distribution $\sigma_z(x)$. The phenomenon that in practice the aligning torque indeed varies in this way is probably due to a combination of several effects. Apart from the cause just mentioned above, the rolling resistance force acting out of the wheel plane (along the actual deformed belt, out of the wheel plane cf. figure 32), may contribute. Another important factor causing the moment to become positive is the fact that the coefficient of friction is not a constant but may depend on the sliding velocity as we shall see later. That means that the coefficient of friction will change (may decrease) in the sliding part of the contact area, which also causes the slight drop in the $F_y(\alpha)$ – curves as has sometimes been found experimentally at high slip values especially on wet roads. The influence of different but symmetric shapes for the vertical force distribution along the x-axis has been theoretically investigated by Borgmann [4]. He finds that, especially for tyres exhibiting a low carcass stiffness, the influence of the pressure distribution is of importance and has, as may be expected, particular effect on the aligning torque at higher values of slip angle $\alpha$.

Many authors adopt the parabolic normal stress distribution in the contact area for purpose of mathematical simplicity, or a uniform (rectangular) distribution.

Figures 34 and 35 show that, when the model parameters are chosen properly, the choice of the type of carcass model hardly influences the results.

5.5. The Brush model

For illustration, we shall present now the theory of steady-state slip with the aid of the simple brush-type tyre model, originally stemming from Fromm. The theory of this section will not consider camber and turning (turnslip) of the wheel. See [23] for an extensive treatment of the brush-model. We refer to figure 36 for a schematic layout of the model. The tyre is equipped with small linear beams (brush elements), some of
which touch the ground and, as a result, will be deformed as a linear beam. Two regions are identified, a leading adhesion region where the contact line (connecting the tips of the brush elements) is straight, and a sliding region where the shear stress follows Coulomb’s law:

\[ \tau = \sqrt{\tau_x^2 + \tau_y^2} = \mu \sigma_z \]

The tyre is moving with speed \( V \), built up from a rolling speed \( V_r \) and a slip speed \( V_s \), with both a lateral and a longitudinal component. The tyre is assumed to move sideways with a slip angle \( \alpha \), in combination with a longitudinal slip \( \kappa \), i.e. we assume the general case of combined slip.

![Figure 36.: The brush-model (according to Fromm)](image)

![Figure 37.: Topview brush model](image)
A topview of the tyre under deflection of the tread elements (the bristles, or brushes) is shown in figure 37.

5.5.1. Displacements in terms of slip and position.
At the leading edge of the contact area, the deformation is still zero. The base and the tip of the tread element coincide. With the tyre moving with speed \( V \) and rolling with rolling speed \( V_r \), the base of the tread is attached to the wheel plane and will move inside the contact area with the rolling speed, say to point B. At the same time, the tip of the tread element will move to point A opposite to speed \( V \). With time-interval \( \Delta t \), this means that the displacement \( w_A \) in the actual contact area along the deformed treads can be written as:

\[
w_A = V \cdot \Delta t
\]

The new positions \( \xi_A \) (tip) and \( \xi_B \) (base) are found from:

\[
\xi_A = V \cdot \cos(\alpha) \cdot \Delta t \\
\xi_B = V_r \cdot \Delta t
\]

from which expressions for the deformation \( e_x \) and \( e_y \) (cf. figure 37) can be derived:

\[
e_x = [V_r - V \cdot \cos \alpha] \cdot \Delta t \\
e_y = V \cdot \sin \alpha \cdot \Delta t
\]

This means that the displacements can be expressed in terms of either the position in the deformed belt situation, \( \xi_A \), or in the undeformed belt co-ordinate \( \xi_B \) as follows:

\[
\begin{pmatrix}
e_x \\
e_y
\end{pmatrix} = \begin{pmatrix}
\frac{V_r - V_x}{V_x} \\
\frac{V_r - V_x}{\tan \alpha}
\end{pmatrix} \begin{pmatrix}
\xi_A \\
\xi_B
\end{pmatrix} = \begin{pmatrix}
\frac{V_r - V_x}{V_r} \\
\frac{V_x}{V_r} \cdot \tan \alpha
\end{pmatrix} \begin{pmatrix}
\xi_B
\end{pmatrix}
\]

The vector of coefficients corresponds to either practical slip and theoretical slip, as defined before. The expressions are of the general form:

\[
\text{displacement} = \text{slip} \times \text{position}
\]

where slip is defined on the basis of either the position \( \xi_A \) with respect to the deformed tyre or the position \( \xi_B \) with respect to the undeformed tyre. This conforms our earlier statement that practical slip quantities are related to the deformed tyre quantities.
whereas the theoretical slip quantities are derived on the basis of undeformed tyre quantities.

The contact area is taken as a square with length 2.a and width 2.b. We assume a parabolic pressure distribution p(x), taken uniform over the contact width 2.b:

\[ \sigma_z(x) = \sigma_{z0} \left[ 1 - \left( \frac{x}{a} \right)^2 \right] \]

with \( \sigma_{z0} \) following from the condition that

\[ F_z = \int_{-b}^{b} \int_{-a}^{a} \sigma(x) dx dy \]

and thus

\[ \sigma_{z0} = \frac{3F_z}{8ab} \]

5.5.2. Adhesion and sliding

We shall now derive expressions for the total displacement \( e = \sqrt{e_x^2 + e_y^2} \) in the contact area, with distinction between adhesion and sliding.

In the adhesion region, it follows that

\[ e = \rho \xi_B = \frac{1}{1 + \kappa} \left[ \sqrt{\kappa^2 + \tan^2 \alpha} \right] \xi_B \]

In the sliding region, assuming Coulomb friction with friction coefficient \( \mu \), the shear stress \( \tau(x,y) \) is bounded by \( \mu \sigma(x) \). The displacement \( e \) is therefore bounded as well, and it follows from the stiffness of the tread, denoted as \( k \)

\[ e = e_{\text{max}} = \frac{\tau(x,y)}{k} = \frac{\mu \sigma_z(x)}{k} = \frac{3 \mu F_z}{8 a^3 b k} (a^2 - x^2) \]

We introduce the tyre parameter \( \theta \) by

\[ \theta = \frac{4 a^2 b k}{3 \mu F_z} \]

resulting in
The break-away point \( \xi_s \) (indicated in figure 37) at which adhesion turns into sliding is found by taking \( e_{\text{max}} \) equal to the deformation \( e \) yielding:

\[
e_{\text{max}} = \frac{\xi_B (2a - \xi_B)}{2a \theta}
\]

Consequently, for \( \rho = 0, \xi_s = 2a \) and the full contact area is in the state of adhesion. With increasing \( \rho \), the break away point \( \xi_s \) moves to a value \( \xi_s = 0 \), attained at \( \rho = 1/\theta \).

In other words, the parameter \( \theta > 1 \) is the reciprocal total slip for which the full contact area is just sliding. Beyond the magnitude \( 1/\theta \), for total theoretical slip, the tyre remains in a state of complete sliding.

In case of pure slip, this situation is reached for either

\[|\alpha| = \alpha_m = \arctan \left( \frac{1}{\theta} \right)\]

or

\[\kappa_m = \frac{1}{\theta - 1}, \text{ in case of driving } (\kappa > 0)\]

\[\kappa_m = -\frac{1}{\theta + 1}, \text{ in case of braking } (\kappa < 0)\]

### 5.5.3. Shear forces

Next, we determine the shear stresses and, from that, the shear force. The shear stresses are found from

\[
\tau(\xi_B) = \tau_a(\xi_B) = k.\xi = -k.\xi_B.\rho \quad ; \quad \text{adhesion region}, \quad \rho = \begin{pmatrix} \rho_x \\ \rho_y \end{pmatrix}
\]

\[
\tau(\xi_B) = \tau_s(\xi_B) = \mu. p(x).\frac{\rho}{\rho} = -k.\varepsilon_{\text{max}}(\xi_B).\frac{\rho}{\rho} \quad ; \quad \text{sliding region}
\]

where it was used that for isotropic tread stiﬂnesses, the shear stress vector has the same (opposite) orientation as the theoretical slip vector.

The shear force is now easily calculated from

\[
F_{\text{shear}} = 2b \int \tau_a(\xi_B) d\xi_B + \int \tau_s(\xi_B) d\xi_B
\]

and the force-components (lateral force, longitudinal force) are obtained from:
\[
F_{\text{shear}} = -\frac{\rho}{\rho} F \equiv -\frac{\rho}{\rho} \sqrt{F_x^2 + F_y^2}
\]

**Remark**

Note that this expression has been used for the empirical Magic Formula to approximate the horizontal contact forces from the pure slip characteristics. We observed earlier that both this approximation based on theoretical slip as the approximation based on practical slip both give satisfactory results.

We easily arrive at:

\[
F = \mu F_z [3/\theta, \rho - 3(\theta, \rho)^2 + (\theta, \rho)^3] ; \rho < 1/\theta
\]

\[
= \mu F_z ; \rho \geq 1/\theta
\]

**5.5.4. Aligning torque and pneumatic trail**

In the same way, one arrives at a closed form expression for the aligning torque \( M_z \):

\[
M_z = 2b \int_0^{2a} \tau_y (\xi_b) (a - \xi_b) d\xi_b = \\
= \frac{\rho^2}{\rho} \mu F_z a [\theta \rho - 3(\theta \rho)^2 + 3(\theta \rho)^3 - (\theta \rho)^4] ; \rho < 1/\theta
\]

In case \( \rho \geq 1/\theta \), \( M_z \) will vanish. Note that this can either be a result of increasing slip angle \( \alpha \) or increasing brakeslip or driveslip \( |\kappa| \). The pneumatic trail follows from the ratio of \( F_y \) and \(-M_z\):

\[
t(\rho) = \frac{1}{3} a \frac{1 - 3(\theta \rho) - 3(\theta \rho)^2 - (\theta \rho)^3}{1 - \theta \rho + (\theta \rho)^2 / 3} ; \rho < 1/\theta
\]

\[= 0 ; \rho \geq 1/\theta
\]

**5.5.5. Tyre characteristics according to the brush model**

The longitudinal and lateral forces under pure slip conditions are shown shown in figure 38, where we have choosen:

\[
k = 2.10^7 \text{ [N/m}^3]\]
\[
b = 0.1 \text{ [m]}
\]

and have used the following approximate relationship between tyre load \( F_z \) and half contact length \( a \) (see also [35]):
\[ a = 0.0011 \sqrt{F_z} \text{ [m]} \]

with \( F_z \) in N.

One observes the side force \( F_y \) to be a monotonous curve, reaching the saturation level \( F_y = \mu F_z \) at \( \alpha = \arctan(1/\theta) \). No slope reversal occurs, as observed in experimental results. A similar behaviour is observed for the longitudinal force. Corresponding trail and aligning torque are shown in figure 39.

![Figure 38: Longitudinal and cornering tyre characteristics, based on the brush model.](image)

![Figure 39: Pneumatic trail & aligning torque vs slip angle](image)

The aligning torque reaches a peak at \( \alpha = \arctan(1/(4.\theta)) \), after which it reduces in absolute size to reach a zero value at \( \alpha = \arctan(1/\theta) \). The aligning torque does not change sign with increasing slip angle in contrast to the earlier presentations of the aligning torque.

The pneumatic trail is a monotonous function in \( \alpha \), starting with a nonzero slope at \( \alpha = 0 \). Again, it tends to zero, which value is reached at \( \alpha = \arctan(1/\theta) \). Its value at vanishing slip angle:
\[ t(0) \rightarrow \frac{a}{3} ; \alpha \downarrow 0 \]

is smaller than normally encountered (around 0.5a).

We note here that the variation of the tyre forces in the tyre load \( F_z \) is different from experimental results. It can be shown that \( F_x \) and \( F_y \) vary proportionally in \( F_z \) (i.e. no degressive relationship). The aligning torque varies proportionally with \( F_z \sqrt{F_z} \).

**5.5.6. Brush model including carcass compliance**

The combined slip characteristics are shown in figure 40 in terms of the \( F_x - F_y \) polar plot and the longitudinal and lateral forces versus longitudinal slip. The polar plot is close to being symmetrical around \( F_x = 0 \). A similar symmetry turns out to be present in the polar plot of \( M_z \) vs. \( F_x \). In order to remove this symmetry (compare with figure 30), one may include the carcass compliance, as indicated in figure 41. The carcass symmetry plane is connected to the undeformed symmetry plane with lateral and longitudinal springs. The longitudinal and lateral forces now contribute to the moment around point C.

*Figure 40.: Polar plot (\( F_x \) vs. \( F_y \)), and interaction between longitudinal forces and side forces, for the brush model.*

*Figure 41.: Including carcass compliance*
It is assumed that the local behaviour in the contact area for the deflected carcass can be described by the brush-model as described above. We have plotted \( M_z \) vs. \( F_x \) for both cases, without and including the carcass compliance, in figure 42.

![Figure 42: Polar plot (\( F_x \) vs. \( M_z \)), for nondeflected carcass (left) and deflected carcass (right).](image)

One observes that the symmetry is lost, but that the behaviour is still different from figure 30. Also the order of magnitude is different (lower) and the Magic Formula data show nonzero values for large longitudinal slip, being due to the residual torque, the nonzero values of the trail for large slip and especially the contribution of the longitudinal force in the aligning torque for combined slip (nonzero moment arms of \( F_x \) contributing to \( M_z \), see the Magic Formula expressions introduced earlier). The brush-model for deflected carcass necessarily leads to zero aligning torque for large slip, corresponding to the right and left ends of the graphs in figure 42. In between, however, the \( F_x \) vs. \( M_z \) graphs may be made more steep by tuning the carcass-compliance, especially by reducing the lateral carcass stiffness.

### 5.6. The brush string model

In [29], the brush model has been combined with a stretched string model, as indicated in figure 43. This model is referred to as the brush-string model, in contrast to the bare string model, applied extensively by Higuchi, see [16]. A bare stretched string model consists of an endless string which is kept under a certain pretension by a uniform radial force distribution, comparable with inflation pressures in real tyres. This string is elastically supported to the wheel centre plane. The deflection in the contact area can be described by two second order differential equations, of the form:

![Figure 43: The combined brush-string model](image)
\[ \sigma_y^2 \frac{\partial^2 u_y}{\partial x^2} - u_y = -\frac{q_y}{c_{cy}} q \]

with lateral deflection \( u_y \), local shear force \( q_y \) (shear stress, integrated over the tyre width), relaxation length \( \sigma_y \), and carcass stiffness per unit length \( c_{cy} \). Likewise in the longitudinal direction. For points of the string outside the contact area, \( q_y \) is taken equal to zero. Under steady state conditions, one may derive for a rotationally symmetric elastic body representing a wheel and tyre rolling over a smooth surface that

\[ V_g = -V(1 + \kappa) \left( \zeta_y + \frac{du_y}{dx} \right) \]

with local sliding speed \( V_g \), theoretical slip \( \zeta_y \) (likewise in \( x \)-direction). This means that, in the adhesion area, the \( x \)-derivative of the local deflection is described by the tyre slip. As observed earlier, in the sliding area, the total shear stress vector is described by the normal tyre stress through Coulomb’s law.

The above equations describe the belt deflection and the contact phenomena, respectively. In fact, this distinction can be made for any model-based tyre handling analysis.

As a result, one is left with a set of equations, that can be solved in a straightforward way. The extension of the bare-string model to the brush-string model leads to slightly more complex equations, but the basis is the same. It involves the inclusion of the tread stiffnesses \( k \), denoted here as \( c_p \).

*Fig.: 44.: Shearforces, sliding speeds for small relaxation lengths, \((\kappa, \alpha) = (0.02, 0.04)\)*
It is of interest to examine the tyre performance for varying treadstiffness where one would expect a ‘brush-type’ behaviour for much lower treadstiffness whereas a ‘bare belt-type’ behaviour is likely to occur for much larger treadstiffness. This has been investigated for the case of small relaxation length.

Results for fixed small (combined-) slipvalues are shown in figures 44 – 45, restricting to the lateral properties only.

The following observations can be made when the treadstiffness is reduced from very stiff (i.e. with a tyre behaving as a stretched string) to very soft (i.e. with a tyre behaving like a rigid wheel with brushes) with the slipvalues and tyreload unchanged.

The total deflection remains more or less unchanged (at least in order of magnitude) whereas the beltdeflection is strongly reduced (and hence the tread deflection strongly increased). The shape of the total deflection over the contact area changes from rather smooth (dominated by beltdeflection) to a shape with a sharp transition between adhesion and rearward sliding region.

For high treadstiffness, two sliding regions are found with the one at the front side of the contact area being very small (in our example about 3 % of the total contact area). With increasing treadstiffness, the transition of the sliding speeds between sliding and adhesion regions becomes less severe. The adhesion area is enlarged with softer treads, at the cost of higher sliding speeds in the rear sliding region. In other words, softer treads increase the cornering and braking potential of the tyre (e.g. wintertyres versus all-season tyres).

### 6. Transient and dynamic performance

For fast maneuvering of the vehicle, the rubber elements in the contact area will not follow the behaviour at the axle instantaneously. This phenomena, known as transient behaviour, can best be described by a first order equation:

\[
\frac{\sigma_y}{V_r} \frac{d}{dt} \xi_{y,area} + \xi_{y,area} = \xi_{y,axle}
\]
for theoretical slips $\zeta_{y,\text{area}}$ and $\zeta_{y,\text{axle}}$ at the contact area and the axle, respectively, relaxation length $\sigma_y$ and rolling speed $V_r$. The relaxation length is found to be well approximated by:

$$\sigma_y = \frac{1}{C_{cy}} \frac{d}{d\alpha} F_y(\alpha)$$

with lateral tyre slipforce $F_y$ in slip angle $\alpha$, and lateral carcass stiffness $C_{cy}$ (describing lateral force vs. lateral deflection for a non-rotating tyre fixed to the ground). The relaxation length is in the order of 2 – 3 times half the contact length, for small slip angle. The above relationship indicates that the relaxation length depends on the slip angle.

A similar discussion can be held for longitudinal transient behaviour, resulting in a similar equation.

### 6.1. Belt dynamics

The transient phenomena are relevant up to an input loading frequency of about 8 Hz. With higher frequencies, belt dynamics may become important. The first vibration modes are related to oscillations of the tyre belt as a rigid ring with respect to the wheel axle. Mainly the side wall stiffnesses are responsible for the tyre behaviour, with these rigid belt modes having frequencies up to about 90 – 100 Hz. Beyond these frequency, flexible eigenmodes start to arise, with belt deflections varying close to harmonically around the tyre circumference. In contrast to transient behaviour, these phenomena (rigid-ring and flexible ring deflections) are referred to as **dynamic tyre behaviour**.

The in-plane and out-of-plane dynamic tyre behaviour has been extensively studied by Zegelaar, Maurice and Schmeitz (see list of references for their theses). For the in-plane behaviour, rigid ring vibration modes are found in [35] for a free tyre for frequencies of about 48 Hz (circumferential, in phase with rim-rotations), 106 Hz (circumferential, out of phase with rim rotations) and 98 Hz (vertical translational modes). The first flexible mode starts to occur at about 92 Hz.

Some standing tyre modes are shown in figure 46 (from [35]).

![Figure 46.: Mode shapes (in-plane) for a standing tyre](image)

For the out-of-plane behaviour, one observes, for a free tyre, rigid ring mode shapes at around 40 – 45 Hz (lateral, camber, yaw mode shapes). The frequencies for modes of a tyre standing on the road appear to be close to the modes of a free tyre.
The relevance of transient behaviour (up to about 8 Hz) and dynamic tyre behaviour (beyond 8 Hz) is schematically shown in figure 47, taken from the MF-Tyre and MF-Swift manual [19]:

The situation of a 2D-road (in-plane) with unevenesses of arbitrary shape is important for the assessment of comfort, ride and durability. Dynamic in-plane behaviour of tyres has been tested extensively by hitting a cleat of certain shape. Such a cleat may be a trapezoidal one. The dynamic response of a tyre hitting a trapezoidal cleat with height of 10 mm is shown in figure 48. Both the time histories for vertical and longitudinal force, and the frequency contents of these signals (auto spectral density) are shown. One observes resonances at 80 – 90 Hz and 40 Hz. The first resonance corresponds to a mix of vertical and out of phase circumferential vibration, the second resonance is related to the in phase circumferential vibration.

*Figure 47.: Main application areas for each tyre model data (from [19])*
6.2. Tyre enveloping properties,
We conclude that, in order to model dynamic tyre behaviour up to about 100 Hz, a rigid ring model for the belt is sufficient to include the relevant belt dynamics. On the other hand, a tyre has enveloping properties such that the load impact at the wheel axle for short wavelength road unevenesses can’t be described by a rigid belt only, and these properties need to be accounted for. In general, there are two alternative ways to describe dynamic tyre performance:

1. with a rigid belt model, but including some filter model to account for the tyre enveloping properties
2. with a full flexible belt model

The first alternative has the advantage of being efficient with respect to the model complexity. The second option has the advantage of not requiring a separate enveloping model at the cost of a relative large number of degrees of freedom. Examples of this second option are FTIRE (see [12] and [13], and RMODK (see [21]).
There are different ways to describe the geometric filter. The idea behind it is that a rigid belt model passing some effective road irregularity profile leads to the same quasi-stationary response at the wheel axle as a real tyre passing the real road profile. Let us consider the situation of a single step. The vertical position of the axle (the effective plane height) for fixed wheel load is depicted in fig. 49.

This **effective plane height** looks like a combination of two quarter sine waves, each with a height equal to the half step height. This behaviour of the wheel axle position can be obtained by pushing a two-point follower system along a single quarter sine wave with height equal to the full step height. The centre point of this system will show the two sine wave behaviour, and the slope of the two-point follower appears to suit well as an effective plane angle, as ‘seen’ by the tyre as a result of its enveloping properties. This two-point follower has a length of about 80% of the tyre contact length, and is therefore described in [35] as the ‘contact patch’. This second single quarter sine wave is called the **basic function** resulting from the specific road profile (an upward step in this case). Such a basis function can be obtained for any road shape. Think of this as starting from a road profile as a superposition of single road steps. Not all of these steps are ‘seen’ by the tyre, but this problem can be accounted for.

As observed by Zegelaar, the basic function (for a step change) primarily depends on the step height and not on the tyre load. On the other hand, the two-point follower length (the ‘shift’ in figure 49) does not depend on the road unevenesses but is...
primarily related to the tyre load. As a result, one may use a dynamic tyre model, based on a rigid ring, where the road profile is replaced by the corresponding basic road profile, and where the shift is continuously adjusted to the changing tyre load. Such a model was developed at the Delft University of Technology, now well known under the name SWIFT (Short Wavelength Intermediate Frequency Tyre model), with links to many standard vehicle dynamics simulation tools such as ADAMS, SIMPACK, MADYMO (see for example references [1] and [19]).

Schmeitz [32] improved the enveloping model by introducing the tandem model with elliptical cams, shown in figure 50. The tandem base length corresponds 80% of the contact length. Both elliptical cams follow the road profile. As a result, the effective plane height follows from the height of the midpoint of the tandem rod. Note that the cams are only allowed to move in vertical direction (vertical sliders).

It was shown in [32] that the shape of the elliptical cams is affected by neither the vertical load, nor the step height (in case of a step change).

6.3. The rigid ring tyre model

The concept of elliptical tyre models has further been extended to multi-track systems with a finite number of parallel tandems. In that way, oblique step inputs can be dealt with, or

![Figure 50: Tandem model with elliptical](image1)

![Figure 51: Full SWIFT rigid ring tyre model](image2)
longitudinal step inputs with a nonzero slip angle. We refer to figure 51 for the full SWIFT rigid ring tyre model. As discussed before, the tyre tread band is modelled as a rigid ring, i.e. a circular rigid body, suspended to the wheel rim through stiffnesses in radial and circumferential direction (in-plane) and in yaw, camber and lateral direction (out-of-plane). The rigid ring is linked to the ground through a contact model, usually based on transient and steady state magic formula descriptions. The residual stiffness is used to ensure that the overall quasi-static tyre stiffness is modelled correctly.

7. Experimental assessment of tyre characteristics
Tyre characteristics are assessed, either on the road or in the laboratory. Tests on the road are realistic, but in general not reproducible. Tests in the laboratory on a drum with diameter in the order of 2 – 2.5 m are reproducible but not realistic regarding the surface conditions. Other possibilities in a laboratory are the inside of a drum (allowing in-door tests for a wet road) or a belt machine. The latter one has a steel belt, kept flat by hydrostatic bearings.

An important problem is the temperature in combination with the fact that the slip conditions during tests to derive tyre characteristic data are, in general, not realistic. A sweep in slip angle between -5 and + 15° within some seconds leads to high temperatures, affecting these characteristics. Combined slip conditions, i.e. a fixed slip angle under braking torque may lead to erroneous tyre characteristics for the same reason.

For an important parameters such as the cornering stiffness, a difference of 30 % has been observed. For the aligning torque, this variation may be even much larger. Carrying out measurements under conditions, comparable with the realistic driving circumstances, these differences can be brought back to the deviations that can be attributed to the change in road curvature. For a 2 m drum, this means an error of about 16 % reducing to about 13 % for a 2.5 m drum.

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Tyre dynamics, tyre as a vehicle component

Part 2.: Driver judgement of tyre handling characteristics

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1. Introduction.

There is no doubt that tyres have a strong impact on vehicle behaviour and on the driver assessment of vehicle performance. This relates to handling, perceived safety and controllability, the amount of effort required to react, course following and straight line stability, etc.

Several of these aspects are known to correlate to some extent with objective indicators such as gains, response times and alike, as obtained from open loop reference tests. But there is more, particularly in relation to the interface between steering system and the driver control and perceived feedback, where these phenomena are still not well understood.

In this document, several studies from the past are discussed focusing on the influence of tyre design parameters on the driver assessment, where both open-loop and closed-loop results are considered. This results in an overview of the discriminating physical tyre parameters examined, the experimental approaches applied, and the output parameters (subjective and objective identifiers) describing the vehicle behaviour.

These studies start with variation in quantities such as tyre pressure, compound, age (effect of tyre wear), geometry whereas vehicle handling simulation studies deal with performance characteristics in terms of cornering stiffness, pneumatic trail, etc. Mathematical studies are suited for interpretation of vehicle handling performance in terms of tyre characteristics (e.g. Magic Formula model data). This means that, in order to use these studies for further investigating the impact of tyre characteristics on driver assessment, relationships between tyre design parameters and performance characteristics are required.

Using a simple simulation model, derived from and validated by realistic vehicle characteristic data, it appears that the complexity of such models (such as the single-track or two-track model) is not appropriate to study the sensitivity of the driver opinion on tyre performance in all its detail.

1.1. Driving task hierarchy

In order to understand the behaviour of a driver-vehicle system under normal or emergency conditions, the role of the tyre is of outmost importance. Tyres keep the vehicle on the road under extreme manoeuvring by the driver in response of unexpected situations, they assist the driver in predicting the performance of his vehicle under such conditions, they confirm him that he is still in control, they inform him about deviations from an intended path through the steering system, that means that they serve to preview and warn for danger ahead, etc. This means that tyres work out on the driver perception and response at different levels. These levels can be considered with reference to the categories of human behaviour and driving task hierarchy as distinguished by Donges [3] and depicted in fig. 1.
At the left of this figure, the classic hierarchy in behavioural categories is shown with distinction between knowledge based behaviour corresponding to the response to unfamiliar situations, rule-based behaviour corresponding to associative response based on selection of the most appropriate alternative according to earlier subjective experience, and skill-based behaviour which can be regarded as an automatic, unconscious reflex. Comparing this classification to the different driving task levels as shown in figure 1, tyres are mainly of relevance at the levels indicated as guidance and stabilisation. The dynamic status of the vehicle involves changes in the input data for the driver, a major part of which is effected by the tyres (steering feel, vibrations, noise, lateral motions, etc.). The driver responds partly at guidance level (such as corresponding to open loop control) and partly at stabilisation level (such as corresponding to closed loop control). The distinction between those two levels depends on the driver and his experience with similar traffic situations. At the lowest level, information is obtained through the dynamics of the vehicle, yielding a perceived friction level, road-wheel contact, road unevenness, resulting cornering and braking resistance on basis of which the driver has to decide, consciously or unconsciously, about safe versus unsafe conditions and the necessary measures to overcome the endangered circumstances. Anticipation of forthcoming situations will improve the driver’s response, and his ability to avoid accidents.

1.2. Driver’s action to emergency situations
Another schematic overview of the driver’s actions to emergency situations has been given by Braun and Ihme and reported by Käppler and Godthelp in [6], see fig. 2. The
three “partners” in any arbitrary traffic situation indicated in the right part of figure 1, i.e. driver, vehicle and environment, are shown in figure 2 as contributors to an experienced level of risk. Such “latent risks” could be effected by poor driving behaviour (like excessive speed), a vehicle deficiency (e.g. low tyre pressure) or changes in the environment (slippery road, poor visibility, dense traffic,...). Reduced safety margins under typical adverse road- and weather conditions have been studied within the DRIVE project ROSES (ROad Safety Enhancement Systems), where not only single causes but also combinations of different hazards have been considered [13]. A sudden event may yield a sharp increase in risk level and, as a consequence, a reduced stabilising tolerance, that is a return to the original risk level. After some reaction time the driver may intervene correctly, he may intervene incorrectly (braking on an icy surface) or he may not respond or respond too late if the accident level has already been reached. Again, it is clear that appropriate information that is based, to a large extent, on tyre performance would help the driver to anticipate risky situations (i.e. reduce the reaction time), whereas the driver-vehicle system performance is crucial to overcome emergency situations.

![Figure 2: Driver response to potentially dangerous situations](image)

### 1.3. Human judgement and automotive industry objectives

The approaches as outlined in the preceding subsections support the conclusions that the tyre-road interface characteristics affect vehicle handling qualities and constitute, through these, a critical factor in the risk reduction potential at critical situations. They contribute to the driver input and the driver’s ability to take appropriate corrective measure to avoid potential crash conditions. As an example one may think of the steering wheel torque feedback, which depends on the non-linear characteristics of tyres and suspension, and which may contribute to the subjective rating of the control behaviour. This example illustrates the interaction of tyre response with other vehicle subsystems, making it more
difficult to obtain a clear understanding of the impact of tyre characteristics on driver judgement and control.

There is yet another more economic reason to look more closely into the driver assessment of tyre characteristics. In the automotive industry there is a strong desire for further improvement of the safety and handling qualities of vehicles, both under normal and extreme operational conditions. As a consequence of this development, vehicle manufacturers presently put increasing demands on the various parts of a vehicle (such as suspension and tyres) in order to guarantee the optimal vehicle handling and safety qualities as envisaged in the vehicle design. Since most of the verification of vehicle performance qualities is based on human judgement, a better understanding of the driver monitoring and assessment process will contribute to an improved vehicle-driver response and a more efficient and effective design process. In particular, this is true when the additional benefits of introducing advanced control concepts as part of new designs are considered, with the objective to improve or maintain the safety of the vehicle under a wide range of driving conditions. One may think of developments related to yaw moment control (ESP) and other slip-control systems to understand that the tyre road interface plays a dominant role here.

The success of critical automotive component design (either related to the tyre/suspension part, or to advanced vehicle control systems) is determined to a large extent by the integrated behaviour of the component-vehicle-driver system. When analysing these developments from an engineering, marketing or business point of view, considering the fact that basically, evaluations are subjective, business risks are implied for a manufacturer when investing in these developments. This situation may be relieved by expanding the knowledge about the human judgement of critical vehicle qualities. Research in the area of human assessment of vehicle performance may lead to a further understanding of the criteria of assessment of an experienced or inexperienced driver in his judgement of vehicle properties.

These considerations lead to the following objectives for this part of module 11 of VERT:

- to contribute in understanding of the impact of tyre characteristics on driver judgement
- to explore the state of the art in the subjective assessment of tyre performance
- to explore potentially appropriate methodologies that could be successfully exploited for further research in this field

2. Human monitoring and tyre characteristics

Several papers have been published in the past on the impact of tyre characteristics on vehicle performance assessment and driver feedback information. These contributions have in common that the sensitivity of selected tyre parameters is investigated using objective or subjective assessment methods where, in some cases, correlations are identified between these open- and closed loop results. Hence, different tyre construction and performance parameters are distinguished (input tyre characteristics), different methodologies are explored related to certain vehicle handling tests, resulting into output
parameters that are either connected to open-loop vehicle performance or subjective driver ratings.

In this order, the previous research results will be treated in these lecture notes, including a discussion on the impact of the various input tyre characteristics on vehicle performance. We start with a concise characterisation of each of the papers.

Roland et al [14] investigated the sensitivity of tyre design (construction, dimensions) and, through that, tyre performance parameters on the vehicle dynamic response. Both manoeuvring and braking were considered. Several test procedures were discussed where some of them were considered not to be appropriate. Correlation between tyre design and vehicle performance appeared to be not clear in many cases, and it was concluded to emphasise directly, in future studies, on tyre performance parameters. Fairlie and Pottinger [4] considered tyres that varied with respect to hardness and hysteresis with the objective to recommend best practice subjective methodologies in order to discriminate between these tyres in terms of suggested handling rating characteristics. They identified the different sources of error in judgement and proposed certain “rules” to minimise these errors. Brindle [1,2] examined the effects of tyre type (radial vs. cross-ply) and tyre dimension (standard vs. low-profile) on vehicle steering and handling and the perception of the driver on these characteristics. Whereas radials were favoured with respect to feelings about safety, security, control in case of emergencies, cross-ply tyres were rated better concerning the “feel” from the road. Brindle concluded that “steering feel” should be further studied from a broader perspective accounting for steering work, driver feedback from the steering, linearity in response, etc. Käppler and Godthelp [6] examined the effect of tyre pressure variations (resulting in different cornering stiffness at front and rear) through both open and closed loop test procedures, as well as subjective rating procedures to verify earlier findings. This work links objective and subjective assessment procedures that should form the basis for further research on the understanding of the impact of tyre characteristics on driver-vehicle performance. Xia and Willis [17] focused on the tyre cornering stiffness and compared different evaluation methods to rank the tyres with respect to vehicle handling performance. In addition to evaluation methods related to single performance parameters such as gain or response time, they considered four-parameter evaluation method attributed to Mimuro [9]. To some extent, this approach can be considered as an extension to the well known two-parameter evaluation method due to Weir and DiMarco [16].

2.1. Input tyre characteristics
The various input tyre characteristics as discussed in literature are summarised in table 1. Distinction is made between construction parameters, geometrical characteristics (dimensions), service parameters such as inner pressure, performance parameters, and ageing of the tyre referring to the effect of age and wear-in procedures on tyre performance characteristics. The tyre construction and geometry imply certain performance characteristics, where the understanding and exploitation of these relationships is one of the main challenges for a tyre manufacturer. With our nowadays tyre models, one is pretty much capable of examining vehicle response as a result of modified tyre performance characteristics. However, by the end of the day, a tyre
manufacturer is faced with the task to manufacture a tyre satisfying such performance requirements. Some comments will be made on these issues later.

<table>
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<tr>
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<th>Additional remarks</th>
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<td>hardness</td>
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<tr>
<td></td>
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<td>cross-ply vs. radial</td>
<td>[1, 14]</td>
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<td>Carcass material</td>
<td>nylon, rayon, polyester</td>
<td>[14]</td>
</tr>
<tr>
<td>Belt material</td>
<td>rayon, Fiberglas, steel</td>
<td>[14]</td>
</tr>
<tr>
<td><strong>Dimensions:</strong></td>
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<tr>
<td>Size</td>
<td>-</td>
<td>[14]</td>
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<tr>
<td>Aspect ratio</td>
<td>standard vs. low profile tyre</td>
<td>[2, 14]</td>
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<td><strong>Service parameters:</strong></td>
<td>incl. mixed conditions (front-rear)</td>
<td>[6]</td>
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<tr>
<td>Inner pressure</td>
<td>-</td>
<td>TIME</td>
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<tr>
<td>Temperature</td>
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<td>Wet vs. dry conditions</td>
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<td>[2, 14]</td>
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<td>[14, 17]</td>
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<tr>
<td>Cornering stiffness</td>
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<td>[17]</td>
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<tr>
<td>Aligning torque</td>
<td>-</td>
<td>[17]</td>
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<td>Pneumatic trail</td>
<td>-</td>
<td>[17]</td>
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<tr>
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<td>$(F_y / F_z)_{peak}$</td>
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<tr>
<td>Braking force coefficient</td>
<td>$(F_x / F_z)_{peak}$</td>
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<td>Wear-in</td>
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</table>

Table 1.: Input tyre characteristics

Finally, we do not pretend to give a full account of the impact of all possible tyre parameters on vehicle-driver performance. For example, the effect of tread design, tyre width, relaxation length etc. are not treated here and open for further investigations.

2.2. Methodologies

Methodologies on the assessment of vehicle performance can be structured as follows:

- **Subjective methodology strategies**
  - **Performance tests**
    Referring to a specific task as determining a maximum speed (lane change), minimum lateral deviations, steering motions (straight lane test), etc.,
  - **Rating scales** (based on a questionnaire) followed by data reduction (PCA: Principal Component Analysis, DFA: Discriminant Function Analysis).
  - **Open questions**, to be considered as additional to the previous two strategies.
• **Objective methodology strategies**
  - Reference Manoeuvres with instrumented vehicles

There doesn’t seem to be a standard test procedure at hand, as illustrated from the tests as encountered in the literature and listed in tables 2 and 3, with some of the performance metrics indicated in the second column.

<table>
<thead>
<tr>
<th>Subjective methodology strategies</th>
<th>Some performance metrics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Realistic driving conditions along mixed routes on public roads, including rural, suburban and motorway roads.</td>
<td>-</td>
</tr>
<tr>
<td>Closed loop straight lane driving test</td>
<td>lateral vehicle position, required steering inputs; both in amplitude and frequency.</td>
</tr>
<tr>
<td>Closed loop double lane change</td>
<td></td>
</tr>
</tbody>
</table>

*Table 2.: Subjective methodology strategies encountered in the literature*

<table>
<thead>
<tr>
<th>Objective methodology strategies</th>
<th>Some performance metrics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random (or swept) steering input test, with frequency range between 0 and 2.5 Hz</td>
<td>phase lags, equivalent time lag, steady state gain; for yaw rate and lateral acceleration</td>
</tr>
<tr>
<td>Step steering input test</td>
<td>response times, overshoot values, TB-factor or “vehicle characteristic”</td>
</tr>
<tr>
<td>Pulse steer input(alternative to random steer test)</td>
<td>-</td>
</tr>
<tr>
<td>Trapezoidal steering input test</td>
<td>peak lateral acceleration, peak yaw and bodyslip angles, peak sideslip angular rate</td>
</tr>
<tr>
<td>Sinusoidal steering input test</td>
<td>similar to above</td>
</tr>
<tr>
<td>Steady state cornering test</td>
<td>understeer factor</td>
</tr>
<tr>
<td>Straight line braking</td>
<td>longitudinal average deceleration</td>
</tr>
<tr>
<td>Braking in a turn</td>
<td>deceleration, body slip angular rate and change in path curvature</td>
</tr>
<tr>
<td>Turning on a rough road</td>
<td>similar to above</td>
</tr>
</tbody>
</table>

*Table 3.: Objective methodology strategies encountered in the literature*

Subjective ratings consist of numerical values, provided by the testdrivers (subjects) for each characteristic from a predefined list, according to a scale of some magnitude. This could be a 5-point scale [1], a 10-point scale [4, 6].

One should be aware that not only the ratings itself are important but also the deviations among the ratings, allowing a distinction between individual assessments by the subjects and assessments with high level of consistency.

Usually, the set of original variables is reduced to a set of so-called Principal Components or factors which can be regarded as orthogonal (statistically independent) to each of the other components (PCA: Principal Component Analysis). Principal Components are weighted linear combinations of the original measured variables. A next step could then be to reduce this set to new linear combinations with maximum
discrimination between two or more clusters (e.g. related to tyres with high and low cornering stiffness). This second step is referred to as Discriminant Function Analysis (DFA). Some researchers skip the PCA-analysis and apply a direct reduction based on the criterium of maximum discrimination, followed by an interpretation towards more independent factors.

Carrying out such PCA-analysis on both open-loop test results and subjective ratings would allow for further correlation studies between the objective and subjective test procedures.

2.3. Assessment of vehicle performance

There is a general understanding that for the evaluation of vehicle handling performance, the steady state gain between yaw rate and steering input, the response times after a step steer, and the equivalent time constant play an important role. Small values of phase lag in both yaw rate and lateral acceleration appear to correlate well with a positive driver judgement of vehicle controllability [17]. In addition, there is evidence that a small phase lag difference between the lateral acceleration and the yaw rate is appreciated by a driver as well. This indicator played an important role in the discussions on four-wheel steering, as well as the criterium of zero sideslip angle. In fact, it was found that, for example at a severe steering manoeuvre, a driver isn’t able to distinguish properly between a nonzero sideslip angle and a delay in yawrate response. The highest correlation was found between subjective rating and the product of steady-state sideslip angle and yaw rate peak time as resulting from the step steer input test. This last combined parameter is usually referred to as the TB factor. Xia and Willis [17] refer to this parameter as the “vehicle characteristic”.

According to the earlier discussion, all of these parameters as derived from some of the tests in table 3, may be further combined into statistically independent factors that may predict certain aspects of driver judgement of vehicle performance.

The equivalent time constant, denoted as $T_{eq}$, is defined by the frequency at which the phase shift between steering angle and yaw rate amounts 45°. This means that the equivalent time constant $T_{eq}$ describes the required driver phase lag compensation and the vehicle’s effective steering bandwidth. It was demonstrated by Weir and DiMarco [16] that the steady state yaw rate gain $G_r$ should not be too high (to avoid nervous behaviour) and not be too low (to avoid excessive steering input). They determined optimal boundaries in

![Figure 3.: Optimal handling boundaries [16]](image)
the \((T_{eq}, G_r)\) plane for expert and typical drivers, which were extended by Godthelp, Ruijs & v. Randwijk [5] for heavy vehicles. These boundaries, as indicated in figure 3, were derived based on a rating of 3.5 and higher on a 10-point scale.

Mimuro [9] extended the idea of multi-parameter evaluation to investigate vehicle handling qualities to a method including four parameters:

- \(G_r\): steady state yaw rate gain
- \(\omega_n\): the yaw rate natural frequency
- \(\xi_r\): the damping ratio for the yaw rate frequency response
- \(\phi\): phase lag of the lateral acceleration frequency response at 1 Hz

These four parameters together form a rhombus, as indicated in figure 4, where the area can be interpreted as a measure for linear vehicle handling potential. This approach has been applied by Xia et al. [17] where the four parameters were obtained from fitting vehicle frequency response functions to the two degree of freedom “bicycle model”, which appeared to work out very well. The most dominant factors turned out to be the natural frequency and the lateral acceleration phase lag, in discriminating between tyres with different cornering stiffness.

![Figure 4: Four parameter presentation [9]](image)

2.4. Subjective characteristics
Let us return to the subjective ratings as mentioned in subsection 2.2. Some of the characteristics have been listed in table 4, with a clarification as far as available from literature. There is no strict order in this list. Again, one observes a lack of standardisation in the type of questions. The conclusion might be drawn that only a thorough analysis of the subjective findings including possibly a correlation with objective results would allow for a clear interpretation of these characteristics, in retrospective.

Clearly, the factors in table 4 are not independent. For example, “linearity in response” and “predictability” are related but formulated at different levels of perception. The same can be said about factors as “handling in general”, “controllability” which are very general and qualitative concepts whereas “amount of effort of steering” or “reaction speed” are much more specific and closer to quantitative parameters as described earlier.
In addition, some factors are restricted to vehicle behaviour up to a moderate level of lateral acceleration (order 0.3 - 0.4 g) such as “linearity in response” whereas other factors are applicable up to the case of extreme manoeuvring such as “perceived safety and security”.

<table>
<thead>
<tr>
<th>Subjective characteristics (in random order)</th>
<th>Predictability</th>
</tr>
</thead>
<tbody>
<tr>
<td>consequences of inattention</td>
<td>number of steering corrections</td>
</tr>
<tr>
<td>Controllability</td>
<td>amount of steering angle</td>
</tr>
<tr>
<td>reaction accuracy</td>
<td>steering sensitivity</td>
</tr>
<tr>
<td>judgement about reaction speed</td>
<td>steering reverse</td>
</tr>
<tr>
<td>amount of steering force</td>
<td>handling in general</td>
</tr>
<tr>
<td>reaction speed (to steering input)</td>
<td></td>
</tr>
<tr>
<td>plowing (controllability vehicle front end )</td>
<td>swingout (controllability vehicle rear end)</td>
</tr>
<tr>
<td>tracking (maintain straight heading)</td>
<td>returnability (to original path)</td>
</tr>
<tr>
<td>perceived safety and security</td>
<td>perceived confidence (predictability)</td>
</tr>
<tr>
<td>sensitivity and lightness of steering</td>
<td>steering qualities in general</td>
</tr>
<tr>
<td>self-aligning strength of steering</td>
<td>vehicle stability</td>
</tr>
<tr>
<td>amount of effort while steering</td>
<td>linearity in response</td>
</tr>
<tr>
<td>amount of perceived feel through steering</td>
<td>amount of steering feel, thought ideal</td>
</tr>
</tbody>
</table>

Table 4.: Some subjective characteristics [1], [4], [6]

Based on maximum discrimination, various researchers have attempted to reduce the set of subjective characteristics to a more independent set of factors, with or without a preceding orthogonalisation step. In this way, Brindle and Wilson [1] concluded that the perceived feel of safety and security (including control in emergency), stability and course following, the effort required to steer the vehicle and the “feel” through the steering system were most appropriate to predict the ranking between different tyre types. Fairlie and Pottinger [4] arrived at steering sensitivity and linearity as most discriminating factors. Käppler and Godthelp [6] resulted at reaction accuracy and the amount of steering wheel angle needed as most consistent subjective characteristics. In contrast, they concluded that the number of steering corrections needed as well as the required steering moment show large variation in driver rating, and therefore should be regarded as more individual assessment.

Finally, Mimuro et. al. [9] gave an interpretation of the “rhombus-parameters” in figure 4:

\[
G_r : \text{handling easiness}
\]
\[
\omega_n : \text{heading responsiveness}
\]
\[
\xi_r : \text{directional damping}
\]
\[
\phi : \text{following controllability}
\]
2.5. Matching tyre characteristics to vehicle performance

In this subsection, the major conclusions are listed from the references mentioned earlier, based on the classification of Table 1, section 2.1.

Differences in tread compound with respect to hardness and hysteresis are well discriminated by the vehicle steering response, indicating how quickly the tyre reacts to a steering input (including both time response and gain). Very low discrimination is found for tracking (how well does the vehicle maintain its course without driver input) and the controllability of the rear end or front end of the car. This result is supported by [6] where it was concluded that different tyre characteristics due to tyre pressure variations have hardly any effect on lateral deviation in straight lane driving.

Different choices for carcass material and belt material yield mixed effects with respect to cornering stiffness, with relative variation in the order of 30% for the desired conditions. No clear relationship was found in [14]. Increased cornering stiffness is normally associated with a higher yawrate gain and shorter response times, and therefore a better subjective evaluation. This is the reason why radial tyres are preferred above cross-ply tyres. It was shown in [14] that this difference between radial tyres and cross-ply tyres might be counteracted by severe (shoulder) wear-in. In addition, cross-ply tyres were reported to give more “feel” through the steering system [1], explained by the occurrence of a higher pneumatic trail. They show a more linear vehicle response in forward speed making extreme conditions better predictable.

These results do not seem to match with the conclusions by Schröder and Jung (reported in [17]) that the effect of the aligning torque on the handling performance is low. Apparently, “feel” should be interpreted here as something different than handling performance. It might be more related to feedback to the driver through the required steering torque, not effecting gains and response times, and not well covered by objective test methods presently in use.

There is evidence that cornering stiffness increases with tyre size and reducing aspect ratio. This last observation is consistent with the preference of drivers for low profile radial tyres with respect to steering feel, vehicle stability, road holding and handling. On the other hand, conventional radials are superior to low profile tyres with respect to steering return strength, rural comfort and rural steering performance. Moreover, conventional tyres tend to yield more “linear” behaviour in yaw rate and lateral acceleration than low-profile tyres.

2.5.1. Impact of service parameters

Regarding service parameters, some observations are listed below with respect to wet surface conditions, the effect of tyre load and the effect of tyre pressure.

Different sources deal with tests on both dry and wet roads. It was specifically concluded in [14] that wet surface testing is a practical and useful approach for research on vehicle response characteristics. Without getting more specific about this conclusion, it seems to apply to braking tests exclusively. For steering tests, reduced ratings are obtained on wet roads making these wet conditions less suitable for judgement of handling performance.

Tyre loads increase the cornering stiffness. The cornering stiffness stabilises beyond a certain load and may even slightly reduce beyond this point. A similar non-linear effect is
well-known regarding the inner pressure. A maximum (optimal) cornering stiffness is obtained for a certain pressure with lower values for smaller pressures (deflected tyre) as well as beyond this pressure value (changing contact patch).

The effect of tyre pressure on vehicle handling judgement has been extensively studied in [6] with mixed pressure conditions (different pressure for front and rear tyres) chosen such that three typical understeer-oversteer conditions resulted:

- standard understeer
- extreme understeer (low front pressure)
- oversteer (low rear pressure)

It was concluded that these tyre pressure variations had hardly any effect on lateral deviation in straight lane driving. In contrast, the required steering activity (magnitude of steering angle as well as the required faster response time) increases with extreme understeer over the entire speed range. A more extreme result was observed in the oversteer situation, however only beyond a certain (critical) speed.

### 2.5.2. Impact of cornering stiffness

So far, we have considered the impact of changing conditions that refer to the tyre-physics or the service conditions. As mentioned earlier, such variation primarily affect tyre performance parameters and, through these, vehicle performance. Below we will focus directly to these last types of relationship.

As noticed before, a higher cornering stiffness correlates with a better handling evaluation by the driver. One should distinguish here between matched tyre conditions at front and rear, and with mixed tyre characteristics.

A higher cornering stiffness in general leads to lower phase lags, both in yaw rate and in lateral acceleration, as well as to a lower phase lag difference between lateral acceleration and yaw rate. In addition, it has been reported to correspond to a lower TB-factor (or “vehicle characteristic”) and a higher yaw rate natural frequency.

For mixed conditions, we refer to the comments on [6] and the observation in [14] that mixed cornering stiffness conditions have impact on the peak lateral acceleration (with trapezoidal steer test, or sinus-steer) indicating a smaller stabilising tolerance (in the sense of figure 2).

### 2.5.3. Effect of tyre wear.

Finally, some comments are made on wear-in procedures and normal tyre wear.

It was observed in [14] that the peak lateral force coefficient is strongly effected by tyre shoulder wear, with opposite results for radials and cross-ply tyres. It illustrates that one should be careful about wear-in procedures.

The evaluation of the understeer-oversteer characteristics of certain mixed tyrepressure conditions as reported in [6] were repeated after one year of normal use. These characteristics appeared to have developed into a more pronounced direction, both for the pressure combination with original understeer performance and the pressure combination with original oversteer characteristics.
3. **The variation of tyre characteristics, a model approach.**

The previous section discussed variation in physical tyre parameters, their effect on tyre performance characteristics and their sensitivity with respect to assessment of vehicle performance. Both links, between tyre design and tyre performance as well as between tyre performance and vehicle performance, are still not well understood.

Tyre performance characteristics can be described using the well known Magic Formula tyremodel, the latest version of which for passenger car tyres is described in [12]. Its basic form is given by

\[
Y(x) = D\{\sin or \cos\}[C \arctan(Bx - E(Bx - \arctan(Bx)))]
\]

with \(Y(x)\) equals either brake force (or driving force) or lateral force in case of the sine version, whereas \(Y(x)\) is related to the pneumatic trail in case of the cosine version. The variable \(x\) denotes the longitudinal or lateral slip. The coefficients \(B, C, D\) and \(E\) are usually described as stiffness factor, shape factor, peak value and curvature factor, respectively.

In order to study the effect of changing of these characteristics on vehicle handling, various User Scaling Factors have been included in the Magic Formula model. Some of these User Scaling Factors are listed below (restricted here to lateral pure slip):

\[
\begin{align*}
\lambda_{Fz0} & : \text{nominal load} \\
\lambda_{\mu} & : \text{peak friction coefficient} \\
\lambda_{Ky} & : \text{cornering stiffness} \\
\lambda_{Cy} & : \text{shape factor} \\
\lambda_{Ey} & : \text{curvature factor} \\
\lambda_{\gamma} & : \text{camber force stiffness} \\
\lambda_{t} & : \text{pneumatic trail}
\end{align*}
\]

For further clarification of these scaling factors, some of the Magic Formula expressions for pure lateral slip are included in this document below in (2) – (7). In (2), expressions of the lateral force and aligning torque are given in general terms, depending on wheel position (expressed by slip angle \(\alpha\), camber angle \(\gamma\), load \(F_z\), pneumatic trail \(t\) and residual torque \(M_{zr}\).

\[
(2) \quad F_y = F_{y0}(\alpha, \gamma, F_z), \quad M_z = M_{z0}(\alpha, \gamma, F_z) = -t. F_{z0} + M_{zr}
\]

The expression (1) is made more explicit in (3), with horizontal and vertical shifts included (absent in (1)) and with the coefficients and their relationship with the scalar factors further clarified (in terms of the tyre load \(F_z\) and nominal load \(F_{z0}\)). The scalar factor for the pneumatic trail is explained by (7).
3.1. Tyre characteristics for varying scaling factors.
Plots for the side force and pneumatic trail are shown in figures 5 and 6 for varying scalar factors \((\lambda_{K_y}, \lambda_{\mu_y})\) (i.e. varying stiffness and friction) and \((\lambda_{K_y}, \lambda_{\mu_y}, \lambda_t)\) (i.e. varying stiffness, friction, trail) respectively.

Each scalar factor is chosen from two extreme values, high (indicated with \(h\)) and low (indicated with \(l\)).

The scalar factors for cornering stiffness and friction will be varied likewise in the next section in full vehicle simulation studies.
4. Tyre sensitivity, simulation studies.
In this section, the behaviour of a vehicle under various driving conditions is studied for different values of the cornering stiffness and friction. Two cases are distinguished here, the “matched case” with the same characteristics at front and rear tyres, and the “mixed case” with different characteristics.

4.1. Varied tyre characteristics
For tyres, some MF-data have been chosen related to a passenger car tyre on a dry road. The User Scalar Factors for cornering stiffness and friction are varied according to the following schedule:

\[ \lambda_{K_y} : 0.5, 0.6, ..., 1.0 \]
\[ \lambda_{\mu_y} : 0.1, 0.2, ..., 1.0 \]

where the cases mentioned above can be expressed as:

- matched case : \( \lambda_{\text{front}} = \lambda_{\text{rear}} \)
- mixed case : \( \lambda_{\text{front}} \neq \lambda_{\text{rear}} \)

4.2. Model description and selected reference manoeuvres
A non-linear vehicle multi-body model has been used in this study with the sprung mass modelled as one (6 dof) rigid body, connected to the unsprung mass with linear springs and dampers. Additional roll stiffness (stabiliser) was included. The tyres were described by the Magic Formula in lateral direction, whereas the vertical behaviour was described by linear springs. The model was validated in the time domain by comparison (and...
tuning) with data from reference handling manoeuvres from real vehicles (high performance passenger car). These manoeuvres included the double lane change, the step steer response and the random steer test.

The impact of varying tyre characteristics will be studied here on the basis of two types of steering input tests: the step steer input test (or J-Turn) to describe the response characteristics of the vehicle to a sudden steer input (response time, overshoot value,...) and the random steer test to generate frequency response data (gain, phase lag,...).

4.3. Results and interpretation
Simulations have been carried out for varying cornering stiffness for both the matched and mixed cases, as indicated above. First, the transferfunctions have been determined. The simulations in the time domain have been carried out for a ramp steer input (approximating the step steer input), growing from 0 to a maximum value within 0.4 sec’s, such that a steady state lateral acceleration of 4 m/s\(^2\) was obtained. With an initial speed of 20 m/s, this corresponds to a steady state bend with radius of 100 m. Lowering the cornering stiffness simultaneously at front and rear tyres leads to lower gain and larger phase lag between steering angle and yaw rate, in contrast to the situation of a reduced lateral stiffness only at the rear tyres. In the latter case, understeer behaviour is reduced and possible oversteer behaviour may result which leads to increased gain at lower frequencies.

These results have been included in a “Weir and DiMarco plot, figure 7 similar to figure 3. Lowering cornering stiffness yields a tendency to “leave” the optimal area for both the matched and the mixed case. In the matched case however, this is due to a required larger steering angle whereas the car responds too violently in the mixed case. In both cases, the large equivalent time lag indicates a slower response to steering input. A stronger steering input (matched case) results into a stronger overshoot. Clearly, the contrary is obtained in the mixed case where in both cases the larger response times are evident. Likewise, a similar effect is obtained for the body slipangle. The roll angle is very much associated to the lateral acceleration and doesn’t show a very significant difference between the matched and the mixed case.
4.3.1. Response times for varying cornering stiffness

The various performance indicators, relating to response time, are shown in figures 8 and 9 for different values of the cornering stiffness scaling factor $\lambda_{K_y}$ for the matched case and mixed case, respectively. Distinction is made between the response time $T_x$ and peak response time $T_{r,max}$, corresponding to the time from the steering ramp until 90% of the steady state value or until the maximum value of variable $x$ is reached, respectively. The variable $x$ indicates yaw rate or lateral acceleration. These times differ in the sense that the stabilising capacity of the car and tyres affects the peak response time. In addition, the time lag between lateral acceleration and yaw rate is shown, as well as the TB factor (vehicle characteristic).

It is interesting to examine the amount of distinction between the three values for the scalar factors, by each of the indicators. Each of the indicators is decreasing with higher cornering stiffness (both in the matched case and the mixed case), normally correlating with an improved driver judgement. This effect is more pronounced in the mixed case, demonstrating a higher sensitivity of cornering stiffness to the subjective assessment of vehicle performance.

However, the response time and peak response time for the lateral acceleration appear to discriminate better here than the corresponding variables for the yaw rate, if one compares the average relative variation per unit change in cornering stiffness. This confirms the results by Xia et al. [17]. Also, the TB-factor distinguishes well between the different values of cornering stiffness and especially the time lag between lateral acceleration and yaw rate shows a good discrimination in both cases.
4.3.2. Effect of friction coefficient on vehicle stability

Next, we have varied the friction levels at front and rear tyres independently. As a result, similar conclusions can be derived regarding the resulting response times, phase lags, gains, etc. A friction level at the rear tyres, exceeded by the friction at front tyres might yield unstable behaviour, that is, the vehicle shows strong oversteer behaviour and high absolute body slip angles are found.

For illustration, the combined effect of reduced cornering stiffness at the rear \((\lambda_{Ky}=0.6)\) and mixed friction levels at front and rear is shown in figure 10, where dark squares indicate unstable behaviour. Restoring the cornering stiffness at the rear to the original value \((\lambda_{Ky}=1.0)\) slightly improves the stability, but the road friction remains to be the dominant factor.

![Figure 10. Stability under the combined effect of reduced friction and modified cornering stiffness at the rear.](image)

5. Discussion

Various studies on the assessment of vehicle performance have been reviewed, especially related to tyre characteristics. In most cases, this assessment is related to indicators that can be well defined by reference manoeuvres such as J-turn, random steer etc. Parameters such as gains, response times and phase lags are able to distinguish well between certain different tyre performance characteristics. Other tyre characteristics such as pneumatic trail do not result in such clear distinction whereas it was discussed in section 2.5 that a higher pneumatic trail might contribute to a better “feel” through the steering wheel to the driver. Moreover, there is some evidence that the ranking over tyres according to this
“feel” does not match the ranking according to the conventional objective indicators such as response times, gains, etc. Another intriguing indicator in this respect is linearity. We concluded that there might be more impact of tyre performance to the driver assessment and performance then what can be described by the present reference manoeuvres. These observations are confirmed by other sources from which it is known that relatively minor changes in tyre design and tyre characteristics may result in significant dissimilarities in subjective driver assessment. Within the limits of human assessment, to a large extent these driver assessments appear to be reproducible. Dominating tyre properties and, additionally, the highly sensitive vehicle suspension/steering system contribute significantly to this assessment-reproducibility. However, it is presently insufficiently clear how such driver judgements are related to vehicle design characteristics.

Many of the studies reviewed above have been carried out on the correlation between driver ratings and objective assessments for dominant vehicle behaviour. However, the situation may be more involved than situations considered in previous studies. It means that relatively small parameter deviations yet have significant influence. Methods to objectively quantify the performance deterioration due to these small parameter deviations are virtually lacking. Further, it is not understood how results of such newly developed objective assessment methods could assist in improving the vehicle design. This demands to develop an understanding of information available to and criteria used by the driver in his judgement process, and of their relation with key variables and parameters of the vehicle. Identifying these key variables and parameters in connection with the information transferred through these variables to the driver constitutes a main research issue, yet to be undertaken.

References


[15] ‘left blank ‘


Tyre dynamics, tyre as a vehicle component
Part 3.: Rolling resistance

Virtual Education in Rubber Technology (VERT), FI-04-B-F-PP-160531
Joop P. Pauwelussen, Wouter Dalhuijsen, Menno Merts
HAN University
October 16, 2007
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1. Introductory comments.
Consider a free rolling tyre, schematically depicted in figure 1. For this free rolling tyre, the following expression is satisfied:

\[ F_x = X \]

with \(-X\) the required longitudinal force (at the axle) to overcome a resistance force \(F_x\) at the contactpatch. This resistance force is referred to as rolling resistance force \(F_R\).

\[ F_R = -F_x \]

In approximation, the rolling resistance force is only depending on the wheelload \(F_z\):

\[ F_R = f_R F_z \]

with rolling resistance coefficient \(f_R\). Because of this rolling resistance force, there must be a driving torque \(M_y\), following from:

\[ M_y = F_x r = F_z r f_R \equiv c F_z \]

with \(r\) the loaded tyre radius. As a result, the resulting wheel load acts slightly in front of the projection of the wheel centre on the contact area (see figure). Consequently, the pressure distribution for a free rolling tyre is nonsymmetric. We observe that the rolling resistance corresponds to the torque \(M_y\).

1.1. How can rolling resistance be explained?
For a rolling tyre, deformation of the tyre material occurs while entering the contact patch. The original undeformed conditions are restored when the deformed area leaves the contact patch again. This process involves energy losses, mainly due to
hysteresis of the rubber material. These losses arise in the tread area, in the belt, in the carcass and in the sidewalls.

An overview of the various contributions in this energy loss is shown in figure 2. These losses together correspond to the rolling resistance force $F_R$.

As a result, the rolling resistance is reduced for:

- less hysteresis in the tyre material
- less deformation of the tyre

This discussion is related for a rigid flat road. For a deformable (compliant) road, such as soil, the resistance is further increased due to additional friction forces between tyre and soil, and the nonelastic deformation of the soil.

The rolling resistance, being in the order of $0.01$ to $0.05$ for a rigid road or hard soil may easily increase to $0.35$ for a wet saturated soil and to more than $1$ for a soft muddy surface. To put it in other words, a wheel on compliant soil attempts to climb out of the pit it is digging itself.

Some areas of the rolling resistance coefficient value are shown in the figure 3 below.

![Figure 3: Rolling resistance values for some road conditions](image)

Other contributions to the overall vehicle resistance include small sliding between road and wheel under normal rolling (adhesion part), aerodynamic drag on the disc (wheel), and friction in the hub.

2. Rolling resistance under driving or braking conditions

The adhesion part is more prominent under driving (tractive force) and braking conditions, leading to higher resistance. The generation of longitudinal forces is always accompanied by some sliding in part of the contact zone. Note that braking and traction also affects the deformation in the contact patch, which may have an impact to rolling resistance, in addition to the occurrence of local sliding.

During small tractive force, the rolling resistance may go down compared to free rolling conditions, up to a level of about $75\% - 85\%$ of free rolling conditions. An example from [3] is shown below in figure 4.
Next to the tyre deflection (deformation), the rolling resistance coefficient $f_R$ also depends on tyre temperature. Both deflection and temperature are affected by the service conditions such as forward velocity, inner pressure and tyre load.

![Diagram of rolling resistant coefficient](image)

*Figure 4.: Rolling resistance under braking and driving conditions*

3. Effect of slipangle and camberangle on rolling resistance

The alignment of the wheel has an impact on rolling resistance. A nonzero slip angle, possibly in combination with some camber will result in a small force acting in the lateral direction, local to the tyre, thus with a component acting in the global forward
direction. Slip angle refers here to the angle between the local wheel velocity at the wheel axis and the intersection of the horizontal plane through the wheel axis and the wheel symmetry plane (perpendicular to the wheel axis orientation). When the wheel alignment forces the tyre to have a nonzero slip angle under normal driving conditions, this slip angle is usually referred to as toe angle. The camber angle refers to the angle between the wheel symmetry plane and the global x-z plane, i.e. it describes the rotation of the wheel symmetry plane in x-direction.

For a slip angle $\alpha$, the contribution of the corresponding local lateral force $F_y = C \cdot \alpha$ (cornering stiffness $C$, small angle assumed) to the forward direction is $F_y \cdot \sin(\alpha) \approx C \cdot \alpha^2$. For a small slip angle (toe angle) of 1° and cornering stiffness being conservatively approximated by $C = 15 \cdot F_z$, one obtains a contribution to the rolling resistance coefficient of 0.02, being in the order of $f_R$ for a dry concrete road with zero slip angle. Hence, a nonzero slip angle has a large impact on rolling resistance, and care should be taken in measurements of $f_R$ to exclude any lateral parasitary forces. For similar reasons, camber contributes to rolling resistance. With a wheel not perpendicular to the ground, a local lateral force arises, which can be discussed in a same way as the lateral force for nonzero slip angle. In addition, under combined camber and sideslip conditions, an aligning torque $M_z$ arises, which has a contribution of $M_z \cdot \sin(\gamma)$ to the driving torque $M_y$.

### 4. Temperature and rolling resistance.

The internal damping of rubber decreases with increasing temperature. As a result, rolling resistance decreases as well. Also, the friction between road and tyre decreases with temperature, resulting in a reduction of the contribution of local sliding in rolling resistance as well. On the other hand, less rolling resistance corresponds to less power dissipation and therefore restricting the temperature rise. Consequently, the decrease of rolling resistance tends to stabilize the temperature of the tyre.

Some results are shown in figure 6 [3] for a start-up process, taking a certain amount of time before equilibrium in temperature and rolling resistance is reached.
Measurement are carried out with a 7.25-13 radial tyre (184/82R13) on a 2.5 m drum with tyre load 4 kN and tyre pressure 1.5 bar.

5. Varying inflation pressure and tyre load.
Increasing the tyre inflation pressure leads to a stiffer belt and therefore a lower rolling resistance. On the other hand, increasing the tyre load leads to more deformation, and therefore to increased rolling resistance. The critical speed increases with lower rolling resistance in these cases. An increase in temperature leads to an increased inflation pressure which lowers the rolling resistance and corresponding heat dissipation, and therefore has a stabilizing effect regarding temperature.

The SAE suggested an empirical formula for the rolling resistance in dependence of inflation pressure $p_i$ [N/m²], forward velocity $v$ [m/s] and tyre load $F_z$ [N]:

$$f_R = \frac{K}{1000} \left( 5.1 + \frac{5.5 \times 10^3 + 90F_z}{p_i} + \frac{1100 + 0.0388F_z}{p_i} \right) v^2$$

The factor $K$ is taken as 0.8 for radial tyres and it is taken as 1 for non-radial tyres.
We have plotted curves in figure 7 expressing the rolling resistance against forward speed for varying tyre load and inner pressure, according to this SAE-expression.

![rolling resistance coefficient](image)

Figure 7.: Rolling resistance for different inflation pressures and tyre load

Other results of the variation of the rolling resistance with inflation pressure are taken from [7] and shown in figure 8. Different inflation pressures are considered on different surfaces. As expected, the effect of increasing the inflation pressure on a soft surface has a more significant effect than on a hard surface such as concrete. Instead of going down, the resistance is increasing with increasing pressure on sand. Lowering the pressure prevent the wheel to ‘dig in’ in the sand which would lead to a rapidly growing resistance.
6. Rolling resistance, varying with forward velocity

The dependency of the rolling resistance on forward velocity \( v \) can be approximated by a higher order formulation, being second order in the SAE-expression, and suggested to be a fourth-order expression in [4], with the second order term neglected being small compared to aerodynamic forces:

\[
f_R = f_{R0} + f_{R1} \frac{v}{100} + f_{R4} \left( \frac{v}{100} \right)^4 ; \text{ } v \text{ in km/h}
\]

The coefficients \( f_{R0}, f_{R1}, f_{R4} \) are shown in figure 9 below as a function of tyre pressure, for three different types of radial (R) tyres:

- **S**: allowable maximum speed of 180 km/h
- **H**: allowable maximum speed of 210 km/h
- **M+S**: tyres, designed for mud and snow (wintertyres)

One observes that the coefficients \( f_{R0} \) and \( f_{R4} \) decrease with inner pressure, which is a result of the fact that the deformation reduces with increasing tyre pressure, and therefore also the rolling resistance. One also observes that the fourth order coefficient \( f_{R4} \) is much smaller for HR tyres than the corresponding values for SR. Clearly, a larger allowable speed requires a lower heat development for the same speed, and this corresponds to a lower rolling resistance and therefore a lower \( f_{R4} \).

We have determined the rolling resistance coefficient for the three tyres treated in figure 9 for nominal pressure and a deviation of + 0.4 bar, based on the mean values of the coefficients \( f_{Ri} \). The results are shown in figure 10. This figure shows the
The integrated effect of tyre pressure, tyre type (S, H) and speed on rolling resistance. Indeed, for high speed, the HR-tyres show the lowest rolling resistance.

The argument about comparison with aerodynamic forces is related to the total resistance for the vehicle. Considering just the rolling resistance, one should include the second order term:

\[ f_R = f_{R0} + f_{R2}v^2, \quad v \text{ in [m/s]} \]

with this expression used in [3], i.e. neglecting the linear term. This is a similar...
expression as introduced earlier, expressing the dependence on tyre load and inner pressure according to SAE. For a radial tyre and conditions with $F_z = 3500$ N, and inflation pressure 1.85 bar, one finds $f_{R0} = 0.0078$ and $f_{R2} = 5.3 \times 10^{-6}$ s$^2$/m$^2$. We have included this graph in blue in the plot with rolling resistance graphs according to the fourth order expression.

One observes that the second order description doesn’t show the sharp increase at large speed, as expected. The higher order approximation should therefore be preferred. The progressive increase of the rolling resistance at higher speed is due to the occurrence of standing waves around the tyre circumference, especially at the trailing edge of the contact area. This will lead to a kind of ‘lift-off’ of the tyre at the rear part of the contact area, with a resulting concentrated contact pressure distribution at the leading part of the contact area. This effect depends on the mass of the tread band. Reducing this mass (resulting in lower rolling resistance) leads to lower centrifugal stiffening and therefore more excessive tyre vibrations. On the other hand, lower mass will increase the natural frequency of the tyre circumferential vibrations, and hence the critical speed. Both effects work against each other. The combined impact on critical speed depends also on the sidewall stiffnesses (being low for radial tyres).

As observed before, the rolling resistance is explained from the torque consisting of the resulting net tyre load times the distance of this resulting vertical force to the wheel centre. This torque is increased in case of standing waves. In the trailing zone of the contact area, the tread has the tendency to reduce the local contact pressure and possible to lift-off from the surface. Consequently, the pressure concentrates more locally at the front part of the contact area, i.e. with the tyre load resultant moving forward. This explains the increased rolling resistance, and strong overheating may take place. This will eventually destroy the tyre beyond a critical speed.

7. Rolling resistance of truck tyres
For truck tyres, the dependency on vehicle speed appears to be more linear, i.e. the factor $f_{R4}$ can be neglected (see [4]). Important for truck tyres is the relationship with tyre load. Increasing load appears to reduce the rolling resistance coefficient, as indicated in figure 11.

![Figure 11. Rolling resistance coefficient vs. tyre load for a truck tyre, from [4](image)](image)
Rolling resistance is very important for heavy goods vehicles. About one third of the energy produced by the engine is used to compensate the rolling resistance. The paper by Popov et. al. confirms that the rolling loss (longitudinal resistance force) is almost linear in the tyre load with the slope slightly increasing with decreasing inner pressure. The rolling loss increases with a decrease in inflation pressure, also leading to a slight increase of the slope (loss versus tyre load). The same tyre deflections correspond with higher rolling loss for higher tyre pressure.

Figure 12 was taken from [5].

Wide single tyres have a lower rolling resistance coefficient compared to conventional truck tyres (about 7 % less).

8. Testing conditions, drum versus flat road.

Results for rolling resistance may be obtained on a flat road or, under controlled testconditions, un a drum in the laboratory with radius usually in the range of 2 – 3 meter. Drum tests may be appropriate for ranking analysis or for investigations in the relative effect of conditions such as load, speed, temperature on rolling resistance. However, the curvature of the drum itself increases the local deformation of the tyre in the contact patch, leading to a larger rolling resistance. With the rolling resistance on the drum $f_{RD}$, drum diameter $D$ and tyre rolling radius $r$, the ratio of the rolling resistances on a flat road and on the drum can be expressed as:

$$\frac{f_{RD}}{f_r} = \sqrt{1 + \frac{2r}{D}}$$

This means that with a tyre radius of 0.35 m and a drum diameter of 2 m, the rolling resistance as determined on a drum should be corrected with a factor 0.8607


Radial tyres normally show a rolling resistance of about 20 % or more lower than bias-ply types, and a higher value of critical speed, see figure 12.

This can be explained by the tyre structure design, leading to less rubber deformation energy for the radial tyre, compared to the bias-ply tyre. This effect is increased by the introduction of low rolling resistance tyres, some years ago, where reduction of 40 % has been claimed with respect to conventional radial tyres, i.e. ending up with half of the rolling resistance of bias-ply tyres.
Other design aspects have an impact to rolling resistance as well, such as the number and orientation of plies, the choice of rubber compounds and the design of treads. Natural rubbers have lower damping compared to synthetic rubbers, leading to lower rolling resistance however at the cost of lower critical speed and smaller lifetime. Patterned treads measurably increase rolling resistance over slicks, because tread rubber bulges and deforms into voids in the tread pattern when the tire bears on the road. This effect is called tread squirm.

![Figure 11.: Rolling resistance coefficient for bias and radial ply tyres (from [6])](image)

10. Rolling resistance on a wet road.

With a significant amount of water on the road, the tyre has to push away this water leading to a larger rolling resistance, depending on the water height \( h \) [mm], the tyre speed \( v \) [km/h] and the tyre width \( b \) [cm]. This resistance will increase with speed up to the level where the full tyre is floating on the water. Beyond this point, the resistance will not increase further with speed. As shown in [2], the effect of speed on the resistance force \( F_{RW} \) can be expressed as:

\[
F_{RW} = A b v^n \ [N],
\]

with exponent \( n \) approximately equal to \( n = 1.6 \) if \( h > 0.5 \) mm  
For \( h = 0.2 \) mm, \( n \) can be approximated by \( n = 2.2 \).  
The value of \( A \) depends on waterheight \( h \). Some results for rolling resistance under aquaplaning conditions are shown in figure 14.
11. Wear and tyre size.
Rolling resistance decreases with wear. Hysteresis losses occur mostly in the tread band. Hence, reducing the tread band material will result in lower resistance.

The two tyre geometrical parameters having an effect on rolling resistance are:

- Tyre radius
- Aspect ratio (section height / tyre width)

Rolling resistance is decreased for a larger tyre radius or a lower aspect ratio (low profile tyres). Hence, smaller tyres have a larger rolling resistance coefficient. However, such tyres are usually used for lighter cars with lower tyre load and therefore lower rolling resistance force.

References.


Tyre dynamics, tyre as a vehicle component

Part 4: Tyre wear

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1 Introduction
Tyre wear has a major impact on both environment and cost. In Europe, 240 million tyres per year are scrapped. The worn tyre carcasses are more and more recycled, which reduces environmental impact of tyre wear. However, the material of the tread that is worn off the tyres ends directly in our environment. Most of this will be as very small particles. With the increase of knowledge on this topic, these particles are considered a severe threat on human health. Besides this environmental (and health) aspect, tyre wear forms a substantial part of the vehicle running cost as well. Although also important for private cars, it is especially for commercial vehicles a factor closely looked after.

Tyre wear is a complex issue, since it conflicts strongly with other properties, e.g. grip and comfort. Besides, it is very strongly influenced by external conditions and therefore difficult to specify. The relationships that apply will be made clear in this section.

First the basic tyre wear mechanisms are explained. Then extraordinary situations of excessive tyre wear are described. In the chapters that follow, tyre wear related aspects of the tyre itself, of the vehicle and of the external conditions are treated respectively.

The next chapter deals with wear on a more general level, e.g. testing, description of wear performance and mileage.
2 Wear mechanisms
Tyre wear has various causes. The complete process can be divided into separate mechanisms, and for each mechanism various actions can be taken to improve the tyre longevity. Each physical mechanism will be described separately, while also attention will be paid to circumstances that can lead to excessive wear.

2.1 Adhesive wear
The mechanism of adhesive wear is based on interaction between materials in the contact area. This is very much a local phenomenon, taking place on small spots where the contact between tyre and road is very intense. Rubber welds to the road, as a result of adhesive force. Because of the movement of the tire, this welding-bond is broken again, leaving particles of the rubber material on the road.

Factors of influence:
- Differences in the adhesion level itself, for various rubbers. The hardness of the rubber influences the adhesive wear: A harder rubber generally has a less intense adhesion with the road.
- Differences in resistance to tearing, for various rubbers. Harder rubber in general has a higher tensile strength. This reduces the loss of material while breaking the bond between tyre and road.
- The road surface has a certain roughness. The deformation of the tire makes it possible to follow this roughness. Concerning the micro roughness of the road, it is the flexibility of the tread material that makes this possible. A softer tread brings a higher percentage of tread in contact with the road. This more intense contact increases wear.
- In general the most effective measure against adhesive wear is lubrication. In the case of tires this occurs on a wet road, by the water actually preventing an intensive contact between the tread and the road. On a dry road, the mechanism itself can hardly be influenced.

2.2 Abrasive wear
Road surface and particles on the road are cutting and grinding material from the tire. A coarse road surface increases this mechanism of tire-wear. There is a significant difference in wear of a tire used on a road polished by use, and on a new road surface, where the road surface irregularities are much sharper.

\[ W = \gamma \rho f s^2 \]  
\[ \text{[Grosch and Schallamach , 1961]} \]

- \( W \) wear of a slipping wheel per unit distance of travel
- \( \gamma \) material constant giving the abrasion of the tread material per unit energy dissipation . (“abradability”)
- \( \rho \) resilience of the wheel
- \( f \) stiffness factor which can be identified with the slope of the sideforce vs. slip angle
- \( s \) slip, which is equal to the slip angle for the low values involved here
The abrasive wear increases strongly with slip, because slip defines the amount of energy put into the contact patch, which is not only considered proportional to the abrasion loss at a given temperature, but it also gives an increase in temperature. A higher temperature increases this kind of wear, since the material becomes softer.

2.3 Erosive wear

Cutting and grinding of particles in a (liquid) stream against the tyre. This mechanism is intensified by the presence of water and sand or other particles on the road, and thus very sensitive to usage conditions. Harder tread materials will better withstand this wear mechanism.

2.4 Ageing

Ageing as a result of high temperature occurs at temperatures above 150º C. This temperature can be reached at very high speed, or with excessive slip, but not in normal usage conditions.

Ageing as a result of ozone or UV radiation only occurs on the outer surface. The resistance depends on the chosen material. Oxidation of the rubber can happen at the outer surface, but also inside the tyre oxidation is possible, when the tire is filled with air. The inner layer should have a good resistance to oxidation.

Deterioration of the rubber material caused by chemicals hardly occurs with normal road tyres during normal use.

2.5 Fatigue

As a result of deformation, microscopic cracks can originate in the rubber material of the tread. By repeating the deformation, these cracks will grow, eventually leading to loss of material.

This wear mechanism is responsible for the major part of wear of a car tyre. It is strongly related to the raw material properties of the tread surface. Also the construction of the tyre has an important role here. The stiffer the tyre is, the less deformation results, thus limiting fatigue. The effect of this wear mechanism is not only reduced by the stiffness of the tyre carcass construction. High tyre pressure also reduces deformation of the tyre, and thereby fatigue.

2.6 Excessive wear

2.6.1 Flat spots

During strong braking, on cars without ABS, complete locking of the wheel can occur. It is obvious that 100% slipping creates high temperature in the contact patch, and a very high wear-rate can occur. During full locking, wear will only
occur on one spot, creating a flat spot on the tyre. When the locking holds on long enough, the complete tread can be worn away, reaching the carcass. Problem with 100% locking is not only the very high wear, but also the concentration of all wear on one location across the tyre-circumference. This flat spot creates extra vibration and noise while driving, decreasing both comfort and traction. On following occurrences of strong braking, locking will often happen on exactly the same spot, because of reduced grip there and un-roundness of the tyre. Often the tyre has to be replaced.

Since ABS is more and more standard on new vehicles, this kind of wear is disappearing.

2.6.2 Physical damage
It is obvious there are many situations where physical damage to the tyre can occur. A few common causes are:

Parking
Hitting the curbstones with high impact can cause damage to the sidewalls of the tyres. With very strongly assisted power steering systems it may also happen that the tyre is crushed against curbstones during turning, or a very high slip percentage is implied on the tyre, increasing wear locally on the tread.

Damaged road
When hitting potholes with high speed, a large share of the impact has to be taken by the tyre. When this impact is too high, which can be the case with high speed or deep potholes, the carcass of the tyre can be damaged.

Objects on the road
Running over objects, for example things lost by other vehicles, can cause damage to the tyres. Sharp objects can lead to punctures, large and hard objects can lead to damage of the carcass.

Often the outer rubber layer is visibly damaged, in other cases only the inner carcass is damaged, which is a dangerous situation. The strength of the tyre is reduced. Locally a lot of extra flexibility is introduced. During driving this generates much local heat, which will break down the tyre further, sometimes leading to a blowout. The intrinsic safety of the tyre is reduced in this way. This kind of damage often shows a radial bulge on the sidewall of the tyre. It is necessary to replace the tyre.

2.6.3 Cupping
Irregular wear patterns, angled pattern blocks, saw-toothed tread-surface are known as cupping. There are various causes for this phenomenon.

- worn shock absorbers
- worn suspension bushes
- loose or worn wheel bearings
- faulty alignment
This kind of wear is very dependent on tyre and car type. In general, very stiff tyres are more sensitive for cupping. Also wide and low-section tyres have a more than average tendency for cupping. In the case of cupping, the stiff carcass makes the tread bounce over the road, instead of continuously following it. Cupping is most common on rear tyres. Reason for this is the lower load of the rear axle on most vehicles. On modern vehicles with complex multi-link rear-axles, the tendency for cupping is increased. These supports have a large amount of joints, which makes them sensitive for play.

Cupping occurs much more on tyres with a block-shaped profile than on V-shaped profiles.

A tyre with this kind of wear often makes an increased amount of noise, which sounds similar to worn wheel bearing.

2.6.4 High temperature wear
When the tyre tread surface reaches a temperature above a critical value, a very high wear rate can occur. The rubber will melt, or be smeared out over the road surface. Normally this high temperature is a result of excessive high slip angle or percentage, possibly combined with high outdoor temperature.

3 The Tyre

3.1 Tyre construction
The construction of the tyre influences tyre wear. A stiffer carcass will lead to lower deformation, resulting in lower slip speeds in the contact area. In general this will reduce wear. This stiffness is defined by the stiffness of the steel belts, polymer cord layer, and rubber stiffness. Both the chosen material and its position and orientation play a role. Since this stiffness is also a very important factor for many other tyre properties, the final choice will be a compromise, in which wear will only be a minor aspect.

3.2 Radial versus diagonal tyres
Since a few decades, only the radial tyre is used for passenger cars. Only in special applications the diagonal tyre is still used. The radial orientation of the cord-layers gives the tyre a much stiffer belt and carcass. Apart from the increased driving performance and lower roll resistance, this also reduces deformation of the rubber, resulting in lower wear. In general the radial tyre has a longer life than the diagonal tyre.

3.3 Compound / material

3.3.1 Tread rubber hardness
Choice of tread material is about the largest compromise in tyre design. Soft material gives the best grip; harder material gives better wear resistance, and lower roll-resistance. The softer tread surface can create a higher adhesion force on the road, leading to higher force in the material. Being softer, the material cannot withstand these forces as well, leading to increased wear. In the VERT section about properties and testing of rubber, more detailed information can be found about wear resistance of the various compounds used.
3.3.2 Silica
Modern tyres make more and more use of silica as a filler. These so called “green tyres” show improved performance both in the field of rolling resistance and wet-grip. The first tyres with silica technology showed a small increase in wear resistance, but with further development, especially focused on better dispersion of the silica material into the tyre compound, a small decrease of wear is expected with silica filled tyres. When we consider the fact that silica-tyres show an increased wet-grip, without making the tread softer, we can see this as a positive contribution in terms of wear.

3.4 Tread pattern

Obviously the thickness of the tread has a major impact on the endurance of the tyre, although in practice there is only little variation between the various tyres. The standard tread profile depth for normal tyres is 8mm. Winter- and off-road tyres generally have a deeper tread, up to 10mm. This gives a better grip on unpaved roads. Sport oriented tyres often have less tread-depth, reduction until 6mm occurs. Less tread depth gives less flexibility, creating more steer-precision.

The shape of the tread can be divided into various categories, each having its own strengths and weaknesses in terms of wear, stability, traction and noise.

- Rib shape
- Lug shape
- Rib-lug shape
- Block shape
- Asymmetric pattern
- Directional pattern

Important in terms of tread wear is the freedom of movement the profile has. The more flexible, the larger the tendency to wear will be, especially fatigue. This makes tyres with a block shaped profile wear faster than other treads. This kind of pattern is mostly seen on off-road and winter tyres, where grip on poor road surfaces is considered more important than wear. The other profiles are more in fashion for road tyres.

3.5 Tyre pressure

There are various effects of tyre pressure on the wear at the contact patch. First there is a relationship between pressure and deformation. A higher pressure will make the tyre stiffer. This will reduce deformation while driving, and thus less heat build-up and wear will occur. Similarly, under-inflation increases tyre-wear.

However, when the tire is made too stiff due to over-inflating, so called vibration wear is caused or increased.

When tyre pressure is significantly higher or lower than its nominal value, the shape of the contact patch will be influenced. An under inflated tyre will mainly use its outer edges to transfer force. The increased force per contact area will lead to increased wear on this section. An over-inflated tyre will have a convex shape,
mainly using its centre section for force transfer. This section will then show increased wear.
4 The Car
The car has various parameters that can influence tyre wear rate, although it is not always realistic to judge these aspects separately. It is the combination of driver, car and tyre that defines the wear rate. First there is car mass. This mass defines the load that has to be transferred by the tyre to the road. Primarily of course in vertical direction, but by this mass also the force needed for longitudinal and lateral movement is dictated.

4.1 Performance
Three relevant areas of car performance can be defined here: engine power, braking power, and cornering performance.

The engine power has to be transferred through the tyres. It is obvious that use of high engine power puts a high deformation and slip angle in the tyres, leading to increased wear.

About the same goes for braking performance, all dissipated power has to be transferred through the tyre, increasing wear. The presence of ABS on a car has a positive effect on tyre wear, since it prevents a very high slip-percentage.

It is difficult to give an unambiguous relationship between tyre wear and cornering performance, since there are two opposite effects. A vehicle that is able to generate high lateral acceleration can transfer large forces through its tyres, increasing wear. On the other hand, such a high-performing suspension setup is able to generate lateral acceleration with a lower slip angle than a less “sporty” setup, which lowers wear.

4.2 Wheel alignment

4.2.1 Camber
An axle construction where the wheel is placed under a small angle in the vertical plane is often used on a car. This camber-angle is used to increase the cornering behavior of the car. Under severe lateral acceleration, when the car rolls, the camber angle ensures the complete contact patch is in contact with the road, the wheels are now in vertical position, improving the ability to transfer force.

Differences between camber angles on front- and rear axles also influence under- or oversteer behavior.

On a straight road, a cambered wheel has most load on its inner shoulder, leading to increased wear here. This is one of the reasons only small camber angles are used, usually not larger than 1°. Modern axle constructions are often aimed at increasing camber angles under lateral acceleration, while limiting static camber. This limits straight road wear, while the camber angle in dynamic cornering condition reduces the slip angle. This has a positive effect on wear.
4.2.2 Toe angle
On most cars, the wheels on one axle are not positioned fully parallel. When the wheels point inwards this is called toe-in, when they are pointed outwards, it is called toe-out. There are various reasons for a car developer to choose a certain setting.

For straight-running stability a small toe-out angle is used. In this way, small disturbances acting on the vehicle will not immediately create a change in direction, because this angle first has to be overcome. Another reason is flexibility in the axle. To prevent excessive toe-out during severe braking, an axle can be given a small static toe-in angle. The opposite is done on the driven axle for acceleration, with a static toe-out angle.

There is a strong relation between wear and this toe angle. When the angle leads to a significant slip angle under steady driving conditions, wear increases very strongly. Excessive toe-angles can destroy a tyre within a few hundred kilometers. Generally, the static toe angle is limited to less than 1°. It is important for tyre durability to regularly check this setting.

4.3 Effects of wear on tyre behavior
During the life of a tyre, its properties will gradually change, which can be noticed in car behavior. Various effects play a role.

Especially under wet conditions, a worn tyre has far less capacity to drain water, while driving. It is obvious that when tyre tread depth wears from 8 to 2 mm, the grooves have less than 25% of the original area available to drain water. This reduces grip in all directions. Much sooner the situation of aqua-planing is reached. In this case the tyre has completely lost its contact with the road, and is floating on water. This makes it almost impossible to generate force, making the vehicle unsteerable. The speed at which aquaplaning occurs can be more than 20 km/h lower for a worn tire, compared to a new one.

The decrease of tread depth also slightly changes other characteristics. Flexibility is reduced. The stiffer tyre uses a smaller slip angle for the same amount of lateral force leading to a more direct steering feel. Loss of flexibility will also conduct more vibration into the car.

The loss of tread material decreases the effective tyre radius. To get an idea of the significance of this change, a calculation can be made for a common tyre, e.g. 195/65-15. The nominal diameter of this tyre in new condition is 322 mm. After the tread has worn from 8 to 2 mm, this diameter is reduced until 316 mm, which is a change of 1.9%. The circumference is of course also reduced by 1.9%. This decrease changes acceleration, fuel consumption and leads to a deviation in speedometer reading.

Tyres that have been significantly overheated can have a changed behavior. The high temperature will evaporate volatile components from the tread compound, making them harder. In general this results in a tyre with less grip. The effect is often visible on the tyre because the tread surface shows some discoloration.
5 Conditions

5.1 Driver
As already indicated in the paragraph about car performance, the driver plays a major role in tyre wear. He defines the amount of energy a tire has to cope with. Especially by preventing excessive slip the wear can be strongly reduced. In practice this is done by limiting strong acceleration and deceleration, and reducing cornering speeds.

5.2 Road
Both the macro and micro surface of the road play a role in tyre wear. In the description of the wear-mechanisms in paragraph 2 the role of the road surface is already mentioned.

For adhesive wear, a smoother surface will create a more intensive contact between tyre and road, and will thus increase this form of wear. Because of the flexibility of the rubber, already on relative rough road surfaces, the tread is able to fully follow the road texture, so the influence of a more smooth road is small.

For abrasive wear, the influence of the road is stronger. A road with a large micro roughness is much more abrasive. Because of this, a new road, with sharp particles, generates more tyre wear.

For erosive wear, the presence of loose particles on the road plays a role. This can be for example sand, or particles that are worn from the road surface.

5.3 Weather
The temperature of the tyre surface is defined by ambient temperature, road temperature, and the amount of energy dissipated in the tyre. Higher temperature makes the compound softer, making it more sensitive for all wear mechanisms.

Rain is also a weather aspect that influences wear. The water film reduces tread surface temperature. This reduction can be between 20 and 30°, having a significant impact on rubber hardness, lowering wear. Another aspect of rain is the water acting as a lubricant, which is a large parameter in adhesive wear. This makes that overall, even taking into account that the water streams can increase erosive wear, tyres wear less under rain condition.

6 Testing, indication and modeling

6.1 Test methods
Tyre manufacturers use various methods for testing their tyres for wear resistance. The decision for a method is based on costs, accuracy and test-time.

A differentiation can be made between real-road tests, and laboratory tests.
Real-road tests are time consuming and expensive. Although it is hard to control all the test circumstances, they are almost by definition realistic, though the scope is of course limited to the circumstances under which the testing is performed. As mentioned in the previous section, many parameters influence tyre wear. Laboratory tests are easier and cheaper to perform, under constant conditions, but it takes more effort and control to have realistic and representative testing conditions. Also the interpretation and extrapolation of the results is more difficult. It is also possible to perform laboratory tests on sample pieces, instead of complete tyres.

With real road tests, the result is usually given in depth loss per kilometer, for the laboratory tests the result is often expressed in mass-loss per kilometer.

6.1.1 Real road tests
To be able to judge the results of these tests correctly it is important to have the conditions monitored, a.o. ambient temperature, rain. A common way to do a test is defining a piece of road that will used as testtrack. Often the test is done with two similar cars, fitted with different tyres. The drivers can be interchanged between the testcars regularly. In this way the influence of car and driver can be excluded, and a very good comparison between the different tyre types can be made.

6.1.2 Drum tests
Drums are a common way to test a tire’s characteristics. They are used for defining the lateral characteristics of a tyre, but can also be used for wear testing. Both internal and external drums are used. An important factor is the diameter of the drum. A too small drum will give the tyre more deformation than in road situations. Also more heat will be generated in this way. Another problem is warming-up of the drum, because of this heat. This can make the test far from realistic. There has to be a defined surface on the drum, since this strongly influences the wear behavior of the tyre.

6.1.3 Dedicated material testers
Instead of doing tests with complete tyres, samples of the tread material can be tested on a dedicated tester. The working principles of this kind of testers are described in the course chapter about rubber compounds.

6.2 Tread wear markers
All road legal tyres have so called tread wear indicators in their pattern. Their depth is between 1.6 and 2.0 mm, depending on the brand and type of the tyre. When the indicator is on the same level as the rest of the pattern this tread depth has been reached, indicating that the tyre should be replaced. This depth is the advice of the tyre manufacturer; legally the minimum depth is 1.6 mm for passenger cars, and 1 mm for heavy duty vehicles. On the sidewall the location of the tread markers is indicated by the text TWI, Tread Wear Indicator.

6.3 Tread wear index
The tread-wear index is a rating that can be used as a comparison between tyres, and gives an indication of the durability of a tyre; a higher number stands for more kilometers. Since the absolute durability is very dependent on the vehicle and the
conditions of use, it is almost impossible to relate the index to a specific amount of kilometers. Also between various manufacturers there can be differences between this correlation.

6.4 Modeling
Any attempt to model tire tread wear can be seen as a two stage process. The first stage is the modeling of the mechanical behavior of the tire and tire tread for selected specific operational and vehicle conditions, which in itself is an analytically difficult process. The second stage is modeling the effect of this mechanical behavior on tread wear behavior. This stage is analytically even more difficult, due to the large effect of numerous external parameters on tire wear. Besides, validating this model is extremely difficult, because it implies that all external parameters having an influence on tire wear be kept at a controlled value. Different pavements, different vehicle operations, different atmospheric conditions, different tread rubber compounds, different tread shapes all have their influence. Additionally, many of them have interactions with others, so it is not a relatively simple matter of superimposing the results for the separate factors. So a simple road test is insufficient to give any level of certainty about the model evaluation. This is not to discourage any attempts at modeling tire wear, but is meant to limit expectations of accuracy, due to the complexity of the tire wear process.
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Tyre dynamics, tyre as a vehicle component

Part 5: Tyre noise

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5.1 Introduction

The matter of tyre noise has two aspects: the effect observed inside of the vehicle and the noise emitted to the environment. The first aspect is very much an issue of vibration transferred from the tyre through the vehicle suspension, and will be treated in the chapter on Comfort. This chapter will treat exterior effects of tyre noise.

Besides the annoyance and health effects [1] caused to people, traffic noise also has a large economic effect. The EU Green Paper of 1996 estimated the cost of traffic noise in 17 European countries to be 0.65% of GDP, roughly comparable to half the cost of road crashes. With the expected ongoing increase of traffic volume in the future, the traffic noise problem is expected to increase as well. Tyre noise is the loudest component of the total noise level of distant cars traveling faster than 50 km/h and trucks traveling faster than 80 km/h [2] (Figure 1). Therefore an increasing pressure can be expected on all parties concerned to reduce tyre noise. This of course includes tyre manufacturers.

![Figure 1. Contribution of various sub sources of highway traffic noise. [3]](image)

5.2 Tyre/Road Noise

At the interface between the road and tyre, several mechanisms create energy which is eventually radiated as sound. These mechanisms will be referred to as “sound generation mechanisms”. The mechanisms that convert the energy to sound and radiate it efficiently will be referred to as “sound enhancement mechanisms” [3].

The figures in the following section are from the Tyre/Road Noise Reference Book by Ulf Sandberg and Jerzy A. Ejsmont [5] and are used with permission from the authors.
5.2.1 Sound Generation Mechanisms

Tread Impact
As the tyre rolls over the pavement, an impact occurs as the tyre tread “blocks” hit the pavement at the entrance of the interface between the tyre and pavement (Fig 2). This impact can be compared to a small rubber hammer hitting the pavement. This impact causes vibration of the tyre carcass. The energy created by this impact can be reduced if both the tread block and the pavement can be made more supple.

Air pumping
Within the contact patch, the passages and grooves in the tyre are compressed and distorted, pumping air in and out (Fig 3). This compression and pumping effect aerodynamically generates sound. The effect can be compared to that of clapping hands creating sound.
Slip-stick
During acceleration and braking, and during cornering, horizontal forces are transferred from the pavement to the tread blocks in the contact patch. Additionally, tyre carcass distortion in the contact patch can add to this effect. If the horizontal forces exceed the limits of friction, the tread block will slip briefly and then re-stick to the pavement (Fig 4). This repeated effect of slipping and sticking will generate both noise and vibration. The effect can be compared to sports shoes squeaking on a gym floor.

![Figure 4. Stick-slip motion of the tread block on the pavement.](image)

Stick-snap (adhesion)
The contact between the tread block and the pavement causes adhesion between them. When the tread block exits the contact patch, the adhesive forces pulls on the tread block (Fig 5). The release of the tread block causes both noise and vibration of the tyre carcass.

![Figure 5. Adhesion between tread block and pavement at exit of contact patch.](image)

5.2.2 Sound Enhancement Mechanisms
These effects seen isolated from the tyre/pavement system are in general not sufficient to create a lot of noise. Some aspects significantly enhance the radiated noise.
Horn effect
The geometry of the tyre above the pavement in combination with the pavement itself creates a horn effect, both at the entrance and exit of the contact zone (Fig 6). Sound created near the throat of the horn will be enhanced by the horn. Studies show [4] that this is not merely a reorientation of acoustic energy (a mirror effect), but an actual amplification of the radiated acoustic power.

Organ pipes and Helmholtz resonators
The tread passages in the contact patch resemble acoustical systems that enhance sound generation. (Fig 7). One mechanism is the pipe resonance, due to standing waves in the groove of the tyre tread. The frequency of the resonant sound is determined by the length of the groove. The wavelength will be twice the groove length if the groove is open at both ends, and four times the groove length if the groove is open at one end only. Another mechanism is the Helmholtz resonance. This effect is similar to the whistle produced when blowing across an open bottle. The volume of air in a cavity will act as a spring, making it possible for the mass of air at the opening of the cavity to resonate.
Carcass vibration
The vibration energy created at the tyre/road interface is enhanced by the response of the tyre carcass. Vibrational waves travel in the tread band, which is the structural element of the tyre located adjacent to the tread blocks. These waves create sound which is radiated from the tyre carcass (Fig 8a). Additionally, the tyre carcass sidewalls near the contact patch vibrate and radiate sound (Fig 8b).

Internal acoustic resonance
The air inside the tyre can also be excited by the different mechanisms. At the natural frequency of the toroidal air volume enclosed within the tyre, the air inside the tyre will resonate (Fig 9). The frequency of this resonance is dependant on tyre and rim size and on the speed of sound in the inflating medium. For passenger car tyres this frequency is around 200-300 Hz. This response of the air inside the tyre can be audible.

5.2.3 Spectrum
The effect of the sound produced on the environment is difficult to capture in a number. Different frequencies are considered more or less irritating by individuals, and in general, the environment itself plays a role in the propagation of the sound, dampening some parts of the sound spectrum quicker than others. The tonality of
the sound is very important, because even when the sound pressure is comparable, one sound can be considered acceptable, the other irritating. In general, peaks in the frequency spectrum are to be avoided. Sadly, most tyre/road noise frequency measurements show a broad but pronounced peak in the range of 700-1300Hz (Figure 10 a and b) [5, 6]. According to Sandberg, many factors play a role in this effect. Due to geometrical parameters like tyre size, tread pattern, road surface texture, most of the previously mentioned sound generation and enhancement mechanisms coincidentally have the strongest effect around 1000 Hz. Besides, A-weighting the sound measurement figures exaggerates this effect due to the higher weight between 1000 and 5000Hz. Of course, this weighting is done to match sound measurements to the sensitivity of human hearing and therefore is an indication of the annoyance of the noise.

Figure 10a and b. Third-octave band spectra obtained in TUG/VTI project for 50 different car aftermarket tyres running on the TUG drum with the ISO replica road surface at 90 km/h, shown A-weighted in 10a (left), and with a linear frequency weighting in 10b (right). [6]

5.2.4 Analyzing/reducing tyre noise

General
What makes tyre noise difficult to grasp, and therefore to reduce, is the large amount of external operational factors, outside the tyre itself, having a strong influence on noise levels. This makes it difficult to compare noise measurements done under different circumstances, and also to extrapolate measurements. Another difficulty is that the different sound generation mechanisms have all been shown to be important for certain tyre/pavement combinations. Therefore, different sound generation mechanisms may dominate the sound level for different circumstances, making it difficult to develop a strategy for reducing noise levels for all cases. Additionally, if the different noise mechanisms are similar in strength, tackling one mechanism barely reduces the overall noise level because other mechanisms will become more dominant. Besides, through the different enhancement mechanisms, it can be difficult to determine the sound generation mechanisms responsible for the noise at a given tyre/pavement combination.
Tyre noise is influenced by the following factors:

External: pavement
speed
load
vehicle acceleration
steer angle

Tyre: tread design
state of wear
construction
material
size
inflation pressure

External factors:

**Pavement**
The pavement itself has a very strong influence on the tyre/road noise level. There can be as much as 14 dB(A) difference between a noisy and a quiet pavement, under otherwise similar circumstances, and up to 9 dB(A) difference for a single pavement type, [3]. Tests done in the Netherlands under the IPG program researching “quiet” pavements [7] show that the noise reduction of a certain type of pavement decreases with around 2 dB in the test duration of less than four years.

In general, experiments show that [3]:
1. Coarser surface textures than 20mm tends to increase noise
2. Surface textures finer than 10mm tends to reduce noise
3. Porosity of the pavement tends to reduce aerodynamic noise above 1500Hz
4. Elastic pavement reduces impact and other mechanical sources
5. Better results tend to occur for “negative” texture (below the surface) than for “positive” texture (above the surface)

In general noise levels increase when roads are wet [8]. This effect is substantial at higher frequencies, but has little effect on overall noise levels.

**Speed**
As shown in Figure 1, vehicle speed has a strong influence on tyre noise levels. For different tyre and pavement types the increase in noise level at higher speeds can differ greatly, though in general the Sound Pressure Level is roughly equivalent to the logarithm of the vehicle speed [5].

**Load**
The influence of tyre load (and tyre pressure mentioned later) on noise levels is not very consistent. The reason for the inconsistency is the great number of possible mechanisms that influence noise production relative to tyre load [5]. In general lower load tends to increase low frequency noise on a rough-
textured surface. Higher load increases high and medium frequency levels, and can decrease noise at lower frequencies. In the A-weighting of noise levels, the higher frequencies generally play the more important role here.

**Vehicle acceleration or braking and steer angle**
Higher horizontal force raises the noise levels. For acceleration and braking more than 10dB increase has been measured [5]. At “normal” cornering, A-weighted levels can be raised by up to 3dB, during severe cornering up to 7dB influence has been measured. In the latter case, often specific frequencies are greatly influenced (tyre squeal).

**Tyre factors**

**tread design**
A well randomized tread pattern limits tonal noise components, and is therefore an important issue, already patented by Michelin in 1929 [6]. “Ribs” are generally quieter than “Lugs”. The frequency spectrum can be changed by changing groove lengths and width and angle, or intersecting grooves. To reduce the effect of air pumping on tyre noise, it is necessary to connect any cavities in the tread profile to the outside air to reduce pressure gradients.

**state of wear**
For the state of wear the same applies as for the tread design, as wearing of the tyres basically changes the tread, in any case the tread depth, thereby influencing the noise levels [9]. Sometimes the pattern also changes with wear, either because of shallow sipes (fine slits) that disappear with wear, or because grooves are wider at the top of the tread than near the bottom. For many tyres, a different rubber compound is used for the top layer of the tread than for the major part of the tread, i.e. to give good initial friction values and long lifetime. When the top layer wears away, tyre noise can change.

**construction**
A stiffer belt generally lowers the noise level. Radial tyres are generally more quiet than bias ply.

**material**
In general, the softer the rubber compound used for the tread, the lower the noise level. Often winter tyres are quieter than summer tyres. Using material optimized for low noise levels could mean a sacrificing other tyre aspects, for instance tyre wear. According to an article in Tyre Technology International [9] there is no conflict detected between low noise and high wet friction or hydroplaning. Research has also shown that low noise and low rolling resistance generally go hand in hand. Soft rubber compounds needed for low noise generally do mean a higher tyre wear. If the lower rolling resistance economically speaking compensates for this effect of shorter lifetime is unclear.
The influence of tyre width and diameter on noise levels is not very straightforward, due to the number of noise generation mechanisms influenced. In general increasing tyre width means increasing noise levels. According to [5] 10 mm tyre width increase roughly mean an increase of 0.4 dB noise increase. This effect becomes smaller as tyre width increases above 200 mm. Doubling tyre width could give a noise increase of 4 dB. For tyre diameter, opposing effects are assumed: larger diameter tyres mean an increasing effect of the air displacement mechanism, and a decrease in the tyre impact mechanism, making the net result difficult to predict.

As mentioned in the paragraph about load, tyre pressure has an effect on noise levels, but not very consistent. In general though, lower tyre pressure tends to increase low frequency noise on a rough-textured surface. Higher tyre pressure generally increases high and medium frequency levels on smooth surfaces, and decreases low frequency levels.

5.2.5 Modeling
Many universities and other research organizations and tyre industries have worked or are working on developing advanced simulation models to describe and explain noise generation, aimed at being able to predict the noise effect of a tyre design change. As can be deduced from the large number of factors influencing tyre noise levels and from the large amount of studies on this subject found on internet, this is a tedious job. The European funded project RATIN, based on modeling various tyre noise generation mechanisms including impact and release noise, groove resonance, air pumping and tyre cavity resonance, meant a large step in the right direction.

5.3 Conclusion
Tyre noise is already an important issue, which will become an even larger problem in the near future. As tyre noise is a result of a complicated interaction between tyre surface, tread blocks, pavement, vehicle, carcass and air volumes it is a challenging task to reduce noise production of tyres. Because of the large number of parameters involved, an extensive simulation model is an important tool in tyre design focusing on noise reduction. Softer rubber, fine and randomized tread, and a stiff belt generally speaking mean lower noise level.
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